DESIGN OF ROBUST CONTROLLER FOR VTOL AIRCRAFT

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CERTIFICATE

It is certified that the work contained in the thesis entitled " Design of Robust Controller for VTOL Aircraft " by " Sqn Ldr G . N . Dwivedi " has been carried out under my supervision .

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ABSTRACT

In this thesis the design of a fixed feedback controller for VTOL (vertical takeoff and landing) aircraft with time varying parameters is considered . The robustness of the controller is guranteed in the entire air speed range of 60 to 170 knots .

The design is based on the Linear Quadratic Regulator theory with prescribed degree of stability . A sufficient condition is stated , which when satisfied gurantees optimality of the nominal optimal control law for perturbed system over the entire range of operation . If this condition cannot be satisfied , then a modified condition is stated . This modified condition when satisfied gurantees stability robustness with reduced phase margin , reduced gain reduction tolerance and reduced sector of nonlinearity as compared to that of optimal controller . These reduced stability margins , however are adequate from practical point of view . A fixed controller for VTOL aircraft is designed which gurantees adequate stability robustness and satisfactory performance over the entire speed range .

The responses of VTOL aircraft with this fixed feedback controller are compared with those of optimal controller , approximate optimal controller and existing fixed feedback controller at various air speeds . The comparision establishes adequacy and superiority of the fixed controller designed here over the other existing ones .

CHAPTER 1

INTRODUCTION

1.1 GENERAL

The aspect of robust controller design for linear systems subject to parameter uncertainity has been an active topic of research in recent years. The published literature on the robustness of linear systems can be viewed mainly fron two perspectives, namely : 1) frequency domain techniques and 2) time domain techniques. The main direction of research in frequency domain has been to extend and generalize the well known classical, single-input single-output (SISO) results to the case of multi-input multi-output (MIMO) systems, using the singular value decomposition [1,2]. In the case of frequency domain results , the perturbations are mainly viewed in terms of gain and phase changes .

One factor which clearily influences the analysis and design of robust controller is the characterization of perturbations. Assuming the nominal plant to be stable, the perturbations can be viewed to take different forms like linear, nonlinear, time-invariant, time-variant, structured and unstructred. Structured perturbations are those for which bounds are known for each parameter whereas unstructured perturbations are those for which only a norm bound on the perturbation matrix is known.

In the present work, attention is focused on structured time-varying perturbations and their effects on the nominal model. Under the

perspective of structured time-varying perturbations some researchers have presented analysis and design procedures for robust controllers .

1.2 REVIEW OF EXISTING WORK

In 1964 Kalman [3] gave frequency domain conditions for the single-input single-output linear quadratic regulators. Similar conditions were obtained for multivariable system by Macfarlane [4] . Using the condition of optimality of single-input single-output linear quadratic regulators , it was shown in [5] that the single-inpuit single-output linear quadratic regulators possess most desirable robustness properties . These are infinite upward gain margin , 50 percent gain reduction tolerance , at least $\pm 60^{\circ}$ phase margin and tolerance to nonlinearities lying in the sector $(\frac{1}{2}, \infty)$. These results were extended to multivariable system in [6] .

However, robustness properties cannot be guranteed when the plant parameters vary from their nominal (design) values. Therefore, when designing a robust controller for a system with uncertain parameters , the designer should ensure satisfactory performance not only for nominal plant parameters but also for a whole set around the nominal parameters. There are basically two ways to satisfy this requirement.

(i) Design a fixed feedback controller where the uncertainity has been considered in the design process.

Or

(ii) design an adaptive controller.

1.2.1 Fixed feedback controllers.

Considering the fixed feedback controller design , Singh and Coelho [7] have designed a nonlinear controller to stabilize the perturbed system.

This design is based on the theory of ultimate boundedness. It is claimed that this controller is such that every system response is ultimately bounded within a certain neighbourhood of the desired nonzero set point.

Horisberger and Belanger [8] present an algorithm to determine an output feedback control gain matrix that yields the largest possible tolerable perturbations such that the closed loop system is stable but there are no explicit bounds on the perturbations reported. Zheng [9] presents a procedure to find the stability regions as a function of parametrs but considers only time-invariant perturbations and reports no procedure for synthesizing the controller. Eslami and Russel [10] also address the same problem but no explicit bounds are obtained. Chang and Peng [11] , Patel and Toda [12] , Patel , Toda and Sridhar [13] give bounds on the norn of the purturbation matrix (which amounts to the radius of sphere in the parameter space and thus can be characterized as an unstructered perturbation case).

In the present work, the emphasis is laid on utilizing the structure of the perturbation matrix and thus the aim is to consider bounds on the individual elements of the perturbation matrix rather than on the norm of the perturbation matrix to design a robust feedback controller.

Leitmann [14] considers a class of linear dynamical system in which system state matrix , system input matrix and input itself are uncertain. He discusses the role of the structure of uncertainity in the robust stabilization of uncertain systems using Lyapunov theory but no explicit bounds on the elements of state and input matrices are given .

Yedavalli [15] and [16] obtains improved measure for stability robustness but considers perturbations in state matrix only. Yedavalli ,

Banda and Ridgely [17] consider the same problem with perturbation in both state matrix as well as input matrix . Yedavalli and Liang [18] and [19] suggested coordinate transformation to reduce conservatism in stability robustness bounds . However, no definite procedure for coordinate transformation has been suggested by them. They obtain the bounds on the structured perturbations of an asymptotically stable linear system to maintain robust stability using a Lyapunov matrix equation solution. Also , special cases of the nominal system matrix have been considered for which the bounds are given in terms of the nominal matrix , thereby avoiding the solution of the Lyapunov matrix equation. Yedavalli [20] introduces stability robustness index. He determines the control effort and stability robustness index for different values of feedback gain matrix by plotting relationship between stability robustness index and control effort. He further selects feedback gain matrix which achieves the largest stability robustness index. The method is based on trial and error.

The results of Leitmann [14] , Thorp and Barmish [21] may be termed structural in nature . It means the uncertainity can not enter arbitrarily in to the state equations. Certain pre-conditions must be met regarding the location of the uncertainity within the system description . Such conditions are referred as matching conditions . Matching conditions are only sufficient for stabilizability and not necessary. That is , there exist systems which fail to satisfy matching conditions and yet are stabilizable.

Barmish and Leitmann [22] , Barmish , Corless and Leitmann [23] , Barmish [24] , Petersen [25] , Chen and Leitmann [26] and Chen [27] make use of generalized matching conditions. Barmish and Lietman [22] suggested decomposition of the so called matching conditions . The resulting

decomposed system has two parts , a matched part and a mismatched part. They define measure of mismatch and show that the effective control is possible as long as it does not exceed a critical threshold . This threshold depends on the feedback gain matrix . Hence , by trial and error for the values of the mismatch threshold and feedback gain matrix , the desired ultimate boundedness of solution is achieved.

Barmish , Corless and Lietman [23] suggest the design of linear controller to stabilize a linear time-invariant nominal plant with parameters uncertainity without any coordinate transformation. Barmish [24] derives necessary and sufficient conditions for quadratic stabilizability of the perturbed system.

Petersen [25] introduces a stronger notion of stabilizability , referred as structural stabilizability via a nominally determined quadratic Lyapunov function. He shows that if the system is to have this stonger property, then the matching condition must necessary be satisfied.

Chen and Leitmann [26] describe a general approach towards the design of controller to ensure practical stability of a perturbed system. By practical stability they mean that the system response is ultimately bounded within a small neighbourhood of the origin. Their results are also applicable to nonlinear systems.

Chen [27] introduces the proper decomposition of uncertainity into matched and mismatched parts. Various conditions based on this decomposition are derived or stated . If these are satisfied, then the overall mismatched uncertain system is practically stable.

Petersen [28] , Petersen and Hollot [29] , Schmitendorf [30] - [32] ,

and Hopkins [33] suggest methods-based on the solution of algebric Riccati equation for stabilization of uncertain systems. Petersen [28] pressents procedure for designing full state observer and feedback controller which stabilizes a given uncertain linear system. The design procedure involves solving of two algebric Riccati equations. The design reduces to a standard Linear Quadratic Gaussian (LQG) design if the system uncertainity is absent. The design suggested by Petersen and Hollot [29] involves the use of quadratic Lyapunov fuction . They describe algorithm for the construction of suitable Lyapunov fuction . Once the Lyapunov function is obtained , it is used to construct the stabilizing feedback controller.

Schmitendorf [30] considers the design of stabilizing controllers for linear systems with time-varying uncertainity. Based on the Lyapunov stability theory , he develops a numerical method for finding a robust controller that asymptotically stabilizes uncertain system. Schmitendorf [31] and [32] also considers the same problem . He deals with the time-varying additive uncertainity entering through the system state matrix and input connection matrix. He presents a method to obtain a linear state feedback controller which gurantees that the system response will ultimately lie in a pre-specified neighbourhood of the origin. In some situations , this neighbourhood can be made arbitrarily small to achive practical stability.

Hopkins [33] shows that if a positive definite solution to a family of algebric Riccati type equations exists then the uncertain system can be stabilized by a linear fixed feedback controller.

Gaurishanker and Zackowski [34] have designed a controller which

involves assignment of eignvalues of the model .

Noldus [35] presents an algorithm to find out the maximal solution of an algebric Riccati equation depending on some design parameter and checking for its positive definiteness . He considers the perturbations in either state matrix or input matrix but not in both.

Zhao and Khargonekar [36] obtain the bounds on the allowable perturbation. The bounds obtained by them are shown to be less conservative than the existing ones.

Keel , Bhattacharya and Howze [37] give algorithm to find the largest stability hypersphere in the parameter space. This is found by adjusting the controller parameters until the prescribed parameter ranges are contained in the stability hypersphere. The stability margin defined are conservative.

Mori and Kokame [38] consider the perturbed system state and input matrices and assume that only the lower and upper bounds on the parameter variations are known. The design does not give any consideration to nominal plant parameters.

All the fixed feedback controllers discussed above meet stability robustness requirement but do not consider performance robustness.

1.2.2 Adaptive controllers.

Considering the adptive controller design. Narendra and Tripathi [39] and Yadav [40] have discussed an adaptive controller design for stabilizing controllers . Though they discuss controller design for VTOL aircraft , the principle is extendable to any system with structured uncertainity. Narendra and Tripathi [39] suggest use of digital computer for

identification of system parameters and updating of feedback gain matrix . Yadav [40] obtained a variable controller by piecewise linear approximation of parameters' variation . Optimal controllers are designed for end values of the linear range. Approximate optimal controller for intermediate values of the linear range is a convex combinaton of optimal controller at end values. However adptive schemes such as those suggested in [39] are complex and costly to implement on actual systems.

Narendra and Tripathi [39] , Yadav [40] , Zheng [41] , Hole [42] , Hole and Mahanta [43] have designed controllers which meet both stability robustness requirement and performance robustness requirement by considering quadratic performance index in their design. Zheng [41] presents the condition on senstivity of optimality or suboptimality to parameter variations. He also constructs the permitted region for parameters such that the perturbed system remains optimal or suboptimal under the variation of the parameters. The possible ways for expanding the size of permitted region are also suggested. Hole [42] has designed robust optimal controller in the state space domain . The performance index chosen is quadratic with a degree of stability α [5]. Fixed pertuabation in system state matrix is considered . A sufficient condition is derived which when satisfied gurantees optimality of the nominally optimal control law for the perturbed system . Hole and Mahanta [43] also consider the same problem . It is possible that the sufficient condition as given in Hole [42] may not exist. Therefore , they derive modified condition which when satisfied gurantees stability robustness with reduced phase margin , reduced gain reduction tolerance , and reduced sector of nonlinearity.

1.3 PROPOSED WORK IN BRIEF

It is important to note here that

(i) Nrendra and Tripathi [39] and Yadav [40] consider an adptive controller design for VTOL aircraft and describe the results related to performance and stability robustness.

(ii) Singh and Coelho [7] , Yedavalli and Liang [18] , Yedavalli [20] , Schmitendorf [30] , Gaurishanker and Zackowski [34] and Keel , Bhattacharya and Howze [37] consider fixed feedback controller design for VTOL aircraft and ensure only stability robustness.

A controller design based on the method presented by Hole [42] and Hole and Mahanta [43] is used to obtain a robust controller for VTOL aircraft in this thesis. The VTOL response of this controller is compared with the controllers as obtained by Narendra and Tripathi [39] ,Yadav [40] and Yedavalli and Liang [18]

As stated earlier the design deals with the structured time-varying perturbations in both system state and input matrices.

1.4 OUTLINE OF THE THESIS

In chapter 2 , a robust controller design with additive perturbations in state and input matrices is described. The performance index chosen is quadratic. A sufficient condition [42] is srated which if satisfied gurantees the optimality of the nominal optimal control law for a perturbed system. If the condition is not satisfied for given perturbations , then a modified sufficient condition [43] is given. This condition when satisfied will ensure stability robustness with reduced phase margin , reduced gain reduction tolerance and reduced sector of nonlinearity.

In Chapter 3 , based on the design technique presented in Chapter 2 , a robust controller for VTOL aircraft is designed and results are compared with the existing controllers .

Chapter 4 concludes the thesis.

CHAPTER 2

DESIGN OF ROBUST CONTROLLER WITH STRUCTURED PERTURBATIONS

2.1 INTRODUCTION

In this chapter , a robust controller design subject to structured perturbations is considered . The design is based on the Linear Quadratic Regulator (LQR) theory with prescribed degree of stability . A sufficient condition is stated which when satisfied gurantees optimality of the nominal control law for the perturbed system over the entire range of operation . If this condition cannot be satisfied , then a modified condition is stated . This modified condition when satisfied gurantees stability robustness with reduced phase margin , reduced gain reduction tolerance and reduced sector of nonlinearity as compared to that of optimal controller.

2.2 PROBLEM FORMULATION

Consider a linear time-invariant (LTI) system , described by the state equations

$$\dot{x} = A\,x + Bu \tag{2.1}$$

where

$x \in R^n$ is the n dimesional state vector ,

$u \in R^m$ is m dimensional input vector ,

$A \in R^{n \times n}$ is an actual (perturbed) plant state matrix ,

$B \in R^{n \times m}$ is an actual (perturbed) plant input matrix and

$x(0) = x_0$ is the initial state vector .

The corresponding nominal (design) system is described by

$$\dot{x} = A_0 x + B_0 u \quad ; \quad x(0) = x_0 \tag{2.2}$$

where

$A_0 \in R^{n \times n}$ is a nominal plant state matrix and

$B_0 \in R^{n \times m}$ is a nominal plant input matrix .

The perturbed state matrix A and nominal state matrix A_0 , perturbed input matrix B and nominal input matrix B_0 are related by

$$A = A_0 + \Delta A \tag{2.3}$$

$$B = B_0 + \Delta B \tag{2.4}$$

where

$\Delta A \in R^{n \times n}$ is the fixed perturbation in state matrix and

$\Delta B \in R^{n \times m}$ is the fixed perturbation in input matrix.

The quadratic performance index to be minimized is given as

$$J = \int_0^\infty e^{2\alpha t} (x^T Q x + u^T R u) dt \tag{2.5}$$

where

$\alpha > 0$ is a real scalar constant.

$Q \in R^{n \times n}$ is a real symmetric positive definite (p.d.) or positive semi-definite (p.s.d.) state weighting matrix and $R \in R^{m \times m}$ is a real symmetric positve definite (p.d) input weighting matrix.

The choice of $\alpha > 0$, in the performance index places the closed loop poles to the left of the line parallel to the imaginary axis and passing through ($-\alpha$, 0) in the complex plane . The physical significance of this is that the disturbances in the system will die out faster than $e^{-\alpha t}$ .The above performance index ensures degree of stability α .

Assumptions :

(a) Pair [A_0 , B_0] is controllable and

(b) Pair [A_0 , $Q^{\frac{1}{2}}$] is detectable .

This yields a unique optimal contol law which minimizes perfomance index (2.5) subject to the state equation constraint (2.2) as given by [5]

$$u^* = -L_\alpha x = -R^{-1} B_0^T P_\alpha x . \tag{2.6}$$

The superscript * denotes optimal quantity throughout the thesis .

In (2.6) P_α is the unique , symmetric positive definite solution of the algebric Riccati equation

$$A_\alpha^T P_\alpha + P_\alpha A_\alpha - P_\alpha B_0 R^{-1} B_0^T P_\alpha + Q = 0 \tag{2.7}$$

$$\text{where } A_\alpha = A_0 + \alpha I . \tag{2.8}$$

When the optimal control law (2.6) is used in nominal system (2.2) , it results in a close loop system

$$\dot{x} = (A_0 - B_0 L_\alpha) x \tag{2.9}$$

and the optimal cost is given as

$$J^* = x_0^T P_\alpha x_0 , \tag{2.10}$$

with a degree of stability α. However, when the nominally optimal control law (2.6) is used on perturbed system (2.1) , it gives rise to sub

optimal performance and the corosponding cost is given by

$$J = x_0^T P x_0 . \tag{2.11}$$

Assuming that the perturbed close loop system

$$\dot{x} = (A - B L_\alpha) x \tag{2.12}$$

is stable.

P is obtained as a unique positive definite solution of the Lyapunov equation

$$\hat{A}_\alpha^T P + P \hat{A}_\alpha + L_\alpha^T R L_\alpha + Q = 0 \tag{2.13}$$

where

$$\hat{A} = A - B L_\alpha \quad \text{and}$$

$$\hat{A}_\alpha = A + \alpha I - B L_\alpha . \tag{2.14}$$

A sufficient condition involving α , A and B matrices is given which when satisfied gurantees optimality of the perturbed close loop system (2.12) in the sense of following definition derived from Kalman [3].

<u>Definition</u> : - In connection with LQR problem , a feedback law is optimal if there exists a positive definite or positive semi-definate state weighting matrix and a positive definite input weighting matrix such that the resulting quadratic cost function is minimized by the feedback law under consideration. It is shown in [5 , pp. 126 - 127] that

$$G(j\omega) = R^{\frac{1}{2}} L_\alpha (j\omega I - A)^{-1} B R^{-\frac{1}{2}} \tag{2.15}$$

is the loop transfer function matrix of the perturbed close loop system (2.12). The following theorem from [42] gives condition for optimality of control law (2.6) for perturbed (actual) system (2.1).

<u>Theorem</u> <u>1</u> : - If the pair [A , B] is controllable and the pair [A , $Q^{\frac{1}{2}}$] is

observable , then the control law (2.6) is optimal in the sense of the above definition for the perturbed system (2.1) or equivalently

$$[I + G (-j \omega)]^T [I + G (j \omega)] \geq I \tag{2.16}$$

holds for all ω , if

$$F_\alpha = \hat{A}^T P_\alpha + P_\alpha \hat{A} + L_\alpha{}^T R L_\alpha \tag{2.17}$$

is atleast negative semi-definite . In (2.16) $[\, . \,]^T$ indicates transpose of the matrix [.] .

When designing a controller with large parameter variations , it may happen that the condition (2.17) cannot be satisfied for any value of α and hence , no control law , which remains optimal throughout the parameter variation range can be obtained . However condiditon on F_α , can still be satisfied by sacrificing phase margin , gain reduction tolerance and sector of nonlinearity of optimal linear quadratic regulator. The modified version of Theorem 1 above , is given in Theorem 3 of Hole and Mahanta [43] . It says that if

$$F_\alpha = \beta (\hat{A}^T P_\alpha + P_\alpha \hat{A}) + L_\alpha{}^T R L_\alpha \tag{2.18}$$

(where, $\beta > 1$, is a scalar constant) is atleast negative semi-definite for some $\alpha > 0$ and $\beta > 1$, then the perturbed system is robustly stable with reduced phase margin of $\cos^{-1} (1 - \frac{1}{2\beta})$, reduced gain reduction tolerance of $\frac{50}{\beta}$ percent and reduced sector of nonlinearity of $(1 - \frac{1}{2\beta} , \infty)$ as compared with the phase margin of $\pm 60^\circ$, gain reduction tolerance of 50 percent , and sector $(\frac{1}{2} , \infty)$ of nonlinearity for optimal linear quadratic regulator .

2.3 STEPS IN DESIGN OF ROBUST CONTROLLER

The design is carried out in the following steps.

1. Choose $\alpha > 0$ and obtain matrix $A_\alpha = A_o + \alpha I$.

2. Solve the algebric Riccati equation

$$A_\alpha^T P_\alpha + P_\alpha A_\alpha - P_\alpha B_o R^{-1} B_o^T P_\alpha = 0$$

for symmetric positive definite solution P_α .

3. Obtain the controller gain matrix L_α as $L_\alpha = R^{-1} B_o^T P_\alpha$.

4. Obtain $F_\alpha = (A - B L_\alpha)^T P_\alpha + P_\alpha (A - B L_\alpha) + L_\alpha^T R L_\alpha$.

5. Check for the sign definiteness of F_α . If F_α is atleast negative semi-definite then the controller L_α will remain optimal throughout the parameter variation range . The designer should obtain minimum value of α , for condition 5 to hold .

6. If condition 5 cannot be satisfied for any value of $\alpha > 0$, then carry out the following additional steps.

7. Steps 1 - 3 remain unchanged . Modify F_α as

$$F_\alpha = \beta\{(A - B L_\alpha)^T P_\alpha + P_\alpha (A - B L_\alpha)\} + L_\alpha^T R L_\alpha.$$

8. Find the minimum value of α and β such that modified F_α as given in step 7 is atleast negative semi-definite.

9. The controller L_α will make the perturbed system robustly stable with reduced phase margin , gain reduction tolerance and reduced sector of nonlinearity as already discussed above .

2.4 CONCLUSION

In this chapter , a robust controller design for perturbed system has

been considered based on the LQR theory with a prescribed degree of stability . An algorithm to obtain robust controller has been given .

CHAPTER 3

ROBUST CONTROLLER FOR VTOL AIRCRAFT AND ITS COMPARISON WITH EXISTING CONTROL LAWS

3.1 INTRODUCTION

In this chapter , VTOL (vertical takeoff and landing) aircraft problem is considered. A robust controller , based on the algorithm given in Chapter 2 has been obtained.

The performance of VTOL aircraft with this controller is compared with the controllers obtained by Narendra and Tripathi [39] , Yadav [40] and Yedavalli and Liang [18].

3.2 AIRCRAFT DYNAMICS

The model used here is the one given by Narendra and Tripathi [39]. This model is obtained by linearization of the system dynamics around a nominal air speed.

The linearized model of the VTOL aircraft in the vertical plane is described by

$$\dot{x} = A\,x + B\,u \tag{3.1}$$

where

A is the (4x4) system state matrix,

B is the (4x2) system input matrix,

x is the 4-dimensional state vector

and

u is the 2-dimensional control (input) vector.

The state variables are

x_1 : horizontal velocity

x_2 : vertical velocity

x_3 : pitch rate

x_4 : pitch angle

The control inputs are

u_1 : collective

u_2 : longitudinal cyclic .

The control input u_1 is used to control the verticalal velocity of the VTOL aircraft by the selection of the desired flight angle. This control also has some effect on the horizonntal velocity . The control input u_2 is used to control the horizontal velocity of the VTOL aircraft.

Narendra and Tripathi [39] consider the nominal air speed as 135 knots. At this nominal air speed (henceforth , the air speed is used as a subscript for matrices) , the system state matrix A_{135} and system input matrix B_{135} are

$$A_{135} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3681 & -0.707 & 1.42 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B_{135} = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.5446 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix} .$$

As the air speed deviates from its nominal value , all the elements in the first three rows of both the matrices will change. Since the most significant change occurs in the elements a_{32} , a_{34} and b_{21} , the rest of the elements can be assumed to remain constant without serious loss of accuracy.

When the air speed changes in the range of 60 knots to 170 knots , the bounds on the parameter values of the variable elements are given by

$$0.06635 < a_{32} < 0.5047$$

$$0.1198 < a_{34} < 2.526$$

$$0.9775 < b_{21} < 5.112 .$$

The nominal air speed in the present design of a robust fixed feedback controller is considered as 132.5 knots.

From the A_{135} and B_{135} matrices described and their parameters variation given in fig. 3.2 - 3.4 , the new nominal system state and system input matrices $A_{132.5}$ and $B_{132.5}$ are obtained for nominal air speed of 132.5 knots as given below.

$$A_{132.5} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.3567 & -0.707 & 1.37666 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}$$

$$B_{132.5} = \begin{bmatrix} 0.4422 & 0.1761 \\ 3.464 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix}.$$

The fixed perturbation matrices as related to the nominal system state and input matrices $A_{132.5}$ and $B_{132.5}$ (when the air speed varies from nominal of 132.5 knots to 170 knots) , are given by

$$\Delta A_{(132.5-170)} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.148 & 0.0 & 1.14935 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\Delta B_{(132.5-170)} = \begin{bmatrix} 0.0 & 0.0 \\ 1.658 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}.$$

The perturbed system state matrix at air speed of 170 knots is

$$A_{170} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.5047 & -0.707 & 2.526 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} .$$

The perturbed system input matrix at air speed of 170 knots is

$$B_{170} = \begin{bmatrix} 0.4422 & 0.1761 \\ 5.112 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix} .$$

The fixed perturbation matrices , related to the nominal system state and input matrices $A_{132.5}$ and $B_{132.5}$ (when the speed varies from nominal value of 132.5 knots to 60 knots) , are given by

$$\Delta A_{(132.5-60)} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & -0.301754 & 0.0 & -1.3002 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$\Delta B_{(132.5-60)} = \begin{bmatrix} 0.0 & 0.0 \\ -2.5671 & 0.0 \\ 0.0 & 0.0 \\ 0.0 & 0.0 \end{bmatrix}.$$

The perturbed system state matrix at air speed of 60 knots is

$$A_{60} = \begin{bmatrix} -0.0366 & 0.0271 & 0.0188 & -0.4555 \\ 0.0482 & -1.01 & 0.0024 & -4.0208 \\ 0.1002 & 0.06635 & -0.707 & 0.1198 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix}.$$

The perturbed system input matrix at air speed of 60 knots is

$$B_{60} = \begin{bmatrix} 0.4422 & 0.1761 \\ 0.9775 & -7.5922 \\ -5.52 & 4.49 \\ 0.0 & 0.0 \end{bmatrix}.$$

The state weighting matrix Q and input weighting matrix R in quadratic performance are chosen as

$$Q = \begin{bmatrix} 0.04 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.25 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{1}{25} & 0.0 \\ 0.0 & \frac{1}{9} \end{bmatrix}.$$

3.3 THE ENVIRONMENT

The time-varying environment in this case is provided by the varying air speed which results in time varying-parameters. Fig. 3.1 shows air speed variation with time while accelarting from 60 knots to 170 knots and also decelerating from 170 knots to 60 knots. Figs. 3.2 - 3.4 show the variation of parametrs a_{32}, a_{34} and b_{21} respectively as function of time.

3.4 NUMERICAL RESULTS

3.4.1 Present Work

The satisfaction of condition (2.17) of Theorem 1 gurantees optimality of the nominal control law for perturbed system for $\alpha > \alpha_{min}$, where α_{min} is the least value of a positive constant which satisfies condition number (2.17).

During the design process , no α_{min} could be obtained for various choises of nominal points in the entire speed range of 60-170 knots . However , some of the robustness margins like phase margin , gain reduction tolerance and sector of nonlinearity offered by Linear Quadratic Regulator (LQR) design satifying condition (2.17) of Theorem 1, are more than those needed in practice, Therefore , it was decided to try LQR design to satisfy condition (2.18) , (taken from Hole and Mahanta [43 , Theorem 3]) . The satisfaction of this condition gurantees infinite gain margin , reduced phase margin , reduced gain reduction tolerance , and reduced sector of nonlinearity depending on the parameter β as given in [43 , Theorem 3] .

For the present problem for perturbation from 132.5 knots to 170 knots

α = 0.8 , β = 1.5 and for perturbation from 132.5 knots to 60 knots

α = 0.6 , β = 1.5 were found to satisfy condition (2.18).

Since the controller for α = 0.8 will also be robust for α =0.6 (corresponding to perturbations from 132.5 knots to 60 knots) , α =0.8 is chosen to obtain the final controller. The feedback gain matrix is obtained as

$$L_\alpha = \begin{bmatrix} 4.1541999 & 0.9204033 & -0.6570093 & -2.7100724 \\ 0.8498025 & -1.3834975 & -0.1989615 & -0.0388060 \end{bmatrix}.$$

The reduced phase margin is $\pm$ 48° , the reduced gain reduction tolerance is 33.33 % and the reduced sector of nonlinearity is ($\frac{2}{3}$, ∞) as compared to the respective values $\pm$ 60° , 50% , ($\frac{1}{2}$, ∞) of optimal controller as given by [6].

When the system response with this controller was obtained (given elsewhere in this thesis), it was observed that as the value of α increases , amplitude of the horizontal velocity reponse goes on decreasing for the same value of control input u_1 . Therefore , a large value of β as 3.2 is chosen . This resulted in minimum value of α as 0.3.

The feedback gain matrix L_α obtained for α = 0.3 is obtained as

$$L\alpha = \begin{bmatrix} 1.524691 & 1.0942967 & -0.4107801 & -1.2023888 \\ 0.426191 & -1.2559518 & -0.1080191 & 0.1608093 \end{bmatrix}.$$

With this controller for α =0.3 , the reducced phase margin is $\pm$

32.5^0 , the reduced gain reduction tolerance is 15.6 % , and the reduced sector of nonlinearity is (0.845 , ∞)

3.4.2 Previous Work (Adptive Controllers)

For the comparison with the existing adaptive control laws given by Narendra and Tripathi [39] and Yadav [40] , the air speed selected are 60 , 115 , 150 and 170 knots.

With the algorithm suggested by Narendra and Tripathi [39] , the feedback gain matrices are optimal at particular air speeds. These are given below

$$L^*_{60} = \begin{bmatrix} 0.92832 & 0.32521 & -0.3371 & -0.84266 \\ 0.05985 & -1.3562 & -0.013739 & 0.44606 \end{bmatrix}$$

$$L^*_{115} = \begin{bmatrix} 0.86156 & 0.96881 & -0.28827 & -0.85001 \\ 0.20344 & -1.2515 & -0.041578 & 0.28537 \end{bmatrix}$$

$$L^*_{150} = \begin{bmatrix} 0.78908 & 1.3286 & -0.2662 & -0.82761 \\ 0.28119 & -1.1402 & -0.076928 & 0.16834 \end{bmatrix}$$

$$L^*_{170} = \begin{bmatrix} 0.73344 & 1.53 & -0.28625 & -0.84113 \\ 0.32184 & -1.0564 & -0.1195 & 0.059357 \end{bmatrix}$$

The approximate optimal controller gain matrices as given by Yadav [40] (henceforth represented by L_{y---})are as given here

$$L_{y115} = \begin{bmatrix} 0.8308301 & 0.9276037 & -0.311675 & -0.841895 \\ 0.1908449 & -1.2063 & -0.05288 & 0.2527085 \end{bmatrix}$$

$$L_{y150} = \begin{bmatrix} 0.768791 & 1.3109462 & -0.295495 & -0.841408 \\ 0.2742052 & -1.110909 & -0.095274 & 0.1296448 \end{bmatrix}$$

3.4.3 Previous Work (Fixed Feedback Controller)

For the comparison with the existing fixed feedback controller given by Yeddavalli and Liang [18] , the air speed selected is 115 knots.

They have considered determination of a state transformation to reduce conservatism in the estimation of robust stability bounds.

The weighting matrices in the performance index are taken as

$$Q = 2\, I_4$$

$$R = \frac{1}{\xi}\, I_2$$

where I_4 and I_2 are identity matrices of dimensions 4x4 and 2x2 respectively.

Using ξ as a design variable , a Riccati-based feedback gain matrix is obtained. The value of ξ for the best result is given as 3.6. The coresponding Riccati-based controller does not satisfy stability robustness test given by them. Therefore , this feedback gain matrix is modified to achieve stability robustness. The resulting feedback gain matrix is given as

$$F_{yd115} = \begin{bmatrix} 0.467 & -0.01388 & -0.539 & -0.806 \\ -0.043 & -0.519 & 0.1899 & 0.731 \end{bmatrix}$$

3.5 COMPARISON WITH EXISTING CONTROL LAWS

Step response comparison

The various responses required for understnding the comparison are plotted in figs. 3.5 - 3.20 . These are as given below ,

Comparison of optimal controller and fixed feedback controller (designed in this thesis) at air speeds of 60 and 170 knots

For 60 knots

(i) $\alpha = 0.8$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.5

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.6

(ii) $\alpha = 0.3$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.7

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.8 .

For 170 knots

(i) $\alpha = 0.8$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.9

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.10

(ii) $\alpha = 0.3$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.11

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.12 .

Comparison of optimal and aproximate optimal controllers and fixed feedback controller (obtained in this thesis) at speeds of 115 and 150 knots

For 115 knots

(i) $\alpha = 0.8$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.13

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.14

(ii) $\alpha = 0.3$ (for this condition , additional plots are given for existing fixed feedback controller given by Yedavalli and Liang [18])

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.15

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.16 .

For 150 knots

(i) $\alpha = 0.8$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.17

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.18

(ii) $\alpha = 0.3$

$u_1 = 1.0$, $u_2 = 0.0$ ------ fig. 3.19

$u_1 = 0.0$, $u_2 = 1.0$ ------ fig. 3.20

3.6 DISCUSSION ON THE VTOL PERFORMANCE WITH VARIOUS CONTROLLERS

The controller should be such that it takes the state x_1 (for $u_1 = 0.0$ and $u_2 = 1.0$) to its desired value in minimum time with least overshoots/undershoots and has a minimum effect on x_2 .

The same phenonmenon is expected when u_1 contols x_2 . i. e. the contoller takes state x_2 (for $u_1 = 1.0$ and $u_2 = 0.0$) to its desired value in minimum time with least overshoots/undershoots and has minimum effect on x_1 .

The response comparison summary is given here between the existing

optimal and approximate optimal controllers and the fixed feedback contoller designed in this thesis.

(i) Condition $\alpha = 0.8$, $u_1 = 1.0$ and $u_2 = 0.0$.

The fixed feedback controller response (figs. 3.5 , 3.9 , 3.13 and 3.17) is seen

better for x_1 for all air speeds,

better for x_2 at air speed of 60 knots ,

inferior for x_2 for all other air speeds and

better for x_3 and x_4 at all air speeds.

(ii) Condition $\alpha = 0.8$, $u_1 = 0.0$, $u_2 = 1.0$.

The fixed feedback controller response (figs. 3.6 , 3.10 , 3.14 and 3.18)

is seen to be

inferior for x_1 at all air speeds ,

on par with optimal controller response for x_2 at all air speeds and

better for x_3 and x_4 at all air speeds.

To improve the response of x_1 for $u_1 = 0.0$ and $u_2 = 1.0$ and that of x_2 for $u_1 = 1.0$ and $u_2 = 0.0$, α was changed from 0.8 to 0.3. The corresponding value of β was obtained as 3.2 . As already discussed above , this results in reduced stability margins but leads to better performance. Moreover , the reduced stability margins are quite acceptable from practical point of view.

The comparison results with the controller designed for $\alpha = 0.3$ are given here.

(i) Condition $\alpha = 0.3$, $u_1 = 1.0$, $u_2 = 0.0$.

The fixed feedback controller response (figs. 3.7 , 3.11 , 3.15 and 3.19) is seen to

remain better for x_1 at all air speeds though it is found to have slightly deteriorated with respect to the responses for $\alpha = 0.8$,

improve further for x_2 at 60 knots ,

improve for x_2 at all other air speeds and

remain better for x_3 and x_4 at all air speeds .

(ii) Condition $\alpha = 0.3$, $u_1 = 0.0$, $u_2 = 1.0$.

The fixed feedback controller response (figs. 3.8 , 3.12 , 3.16 and 3.20) is seen to

improve for x_1 at all air speeds ,

remain on par with optimal controller response for x_2 at all airpeeds and

remain better for x_3 and x_4 at all air speeds.

The response comparison summary between the optimal controller and existing fixed feedback controller is given here

(i) Condition $u_1 = 1.0$ and $u_2 = 0.0$.

For existing feedback controller

x_1 shows large deviation (approximately two times of optimal controller) ,

x_2 shows large overshoot/undershoot and large deviation and

x_3 and x_4 responses are comparable but the settling time is more.

(ii) Condition $u_1 = 0.0$ and $u_2 = 1.0$.

For existing feedback controller

x_1 shows large deviation ,

x_2 also shows large deviation and

x_3 and x_4 show the same settling time , however , the positions of undershoots and overshoots are interchanged.

These comparisons show that

(i) the responses of designed fixed feedback controller are closer to the optimal controller responses .

(ii) the responses of existing fixed feedback controller are far inferior as compared to optimal controller responses and hence , are inferior to the designed fixed feedback controller resposes .

In the existing fixed feedback controller the consideration is given only to stability robustness . The performanse robustness was not taken into account when the feedback gain matrix was modified to achive stability robustness . The fixed feedback controller designed in this thesis gives consideration to performance robustness while maintaining stability robustness. This has resulted in a vastly improved performance as already pointed out above.

3.7 CONCLUSION

In this chapter , the fixed feedback controller for VTOL aircraft was obtained based on the method discussed in Chapter 2 .

The designed controller responses are closer to optimal controller

responses for entire air speed range of 60 - 170 knots . In some cases it is observed that the designed controller responses are even better than the optimal controller responses . Also , the designed controller responses are much better than the fixed feedback controller obtained by Yedavalli and Liang [18].

CHAPTER 4

CONCLUSIONS

The problem of designing a robust controller for VTOL (vertical takeoff and landing) aircraft is considered in this thesis . A fixed feedback controller based on the Linear Quadratic Regulator theory with prescribed degree of stability has been designed at a nominal speed of 132.5 knots .

The controller designed though doesnot posses the performance and stability robustness properties of the optimal controller , the reduced phase margin , reduced gain reduction tolerance and reduced sector of nonlinearity are adequate from practical point of view .

The responses of controller designed are found to be quite satisfactory for various air speeds over the entire range of operation . These are comparable to those of optimal controller and much better compared to those of the existing fixed feedback controllers .

The controller gain matrix being constant , doesnot require use of costly and complex digital computer for parameter estimation as has been suggested by previous designer . Also , there is no need for updating of feedback gain matrix as is the case with gain scheduling controllers . The controller is easy and straight forward to implement on actual system .

The selection of the range of air speed variation from 60 knots to 170 knots , enables this controller to be used in practical environment .

The developments in aviation technology and advances in the VTOL

aircraft capabilities increase the importance of the simple controllers . This study will help in such cases .

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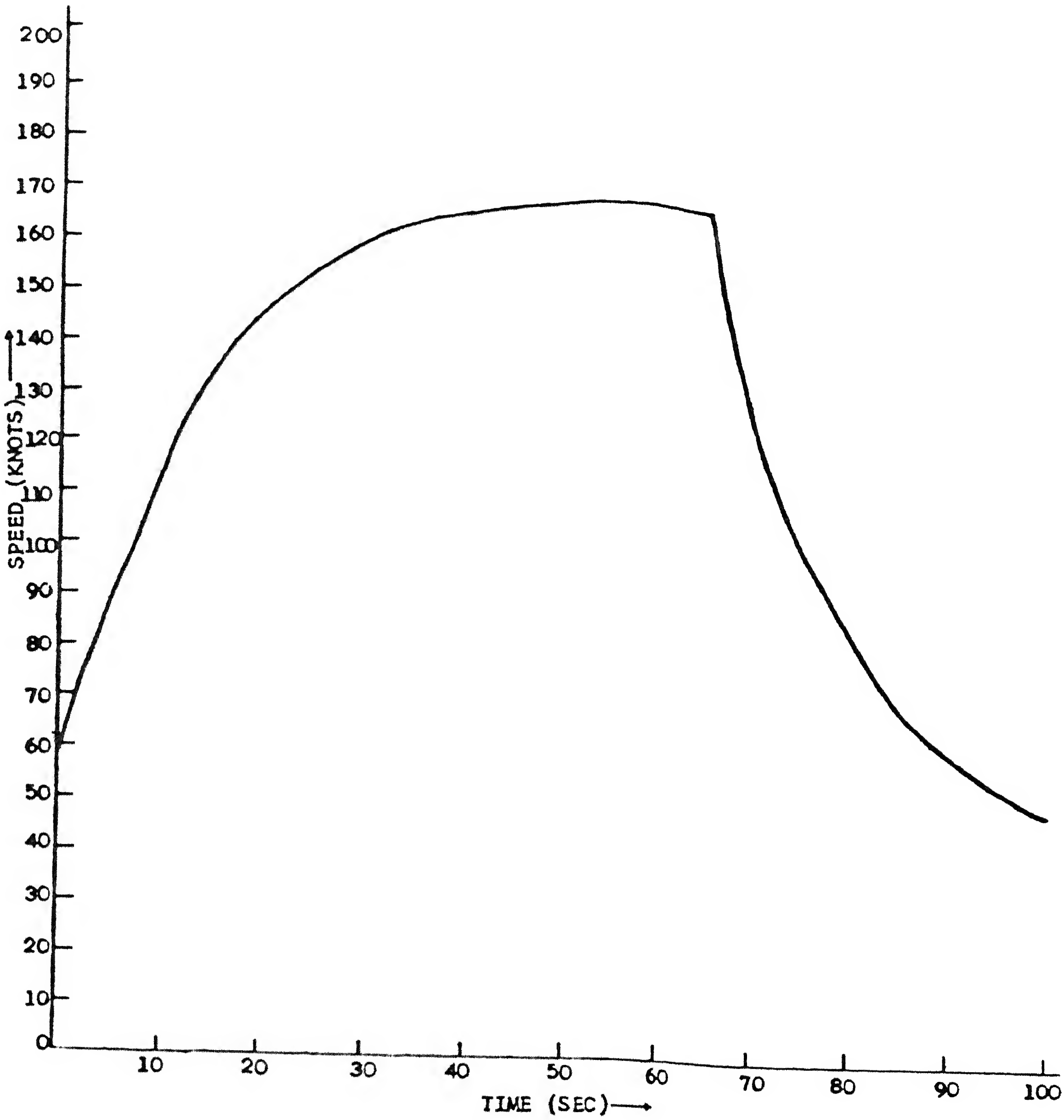


FIG. 3.1 SPEED vs TIME PLOT

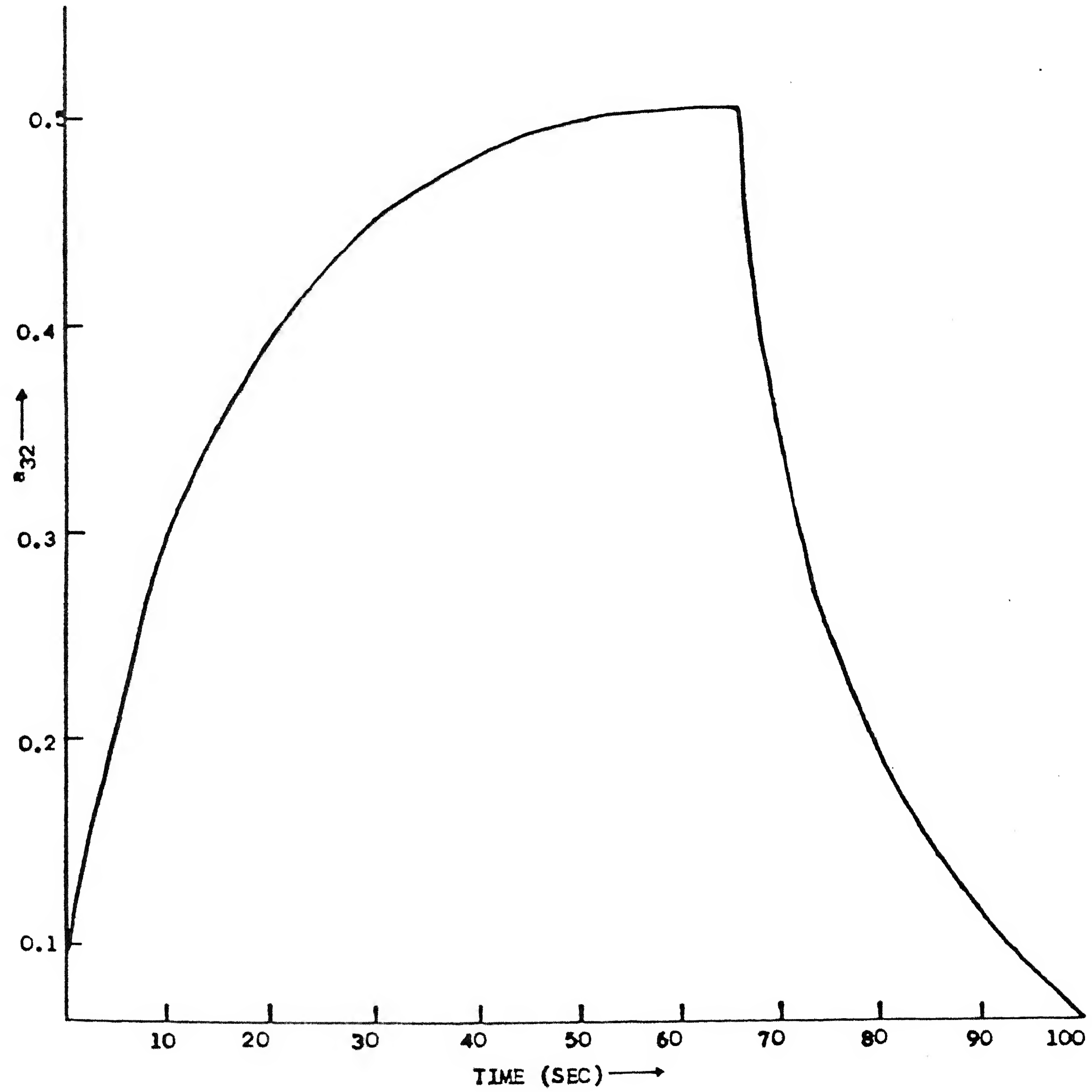


FIG. 3.2 PARAMETER a_{32} vs TIME PLOT

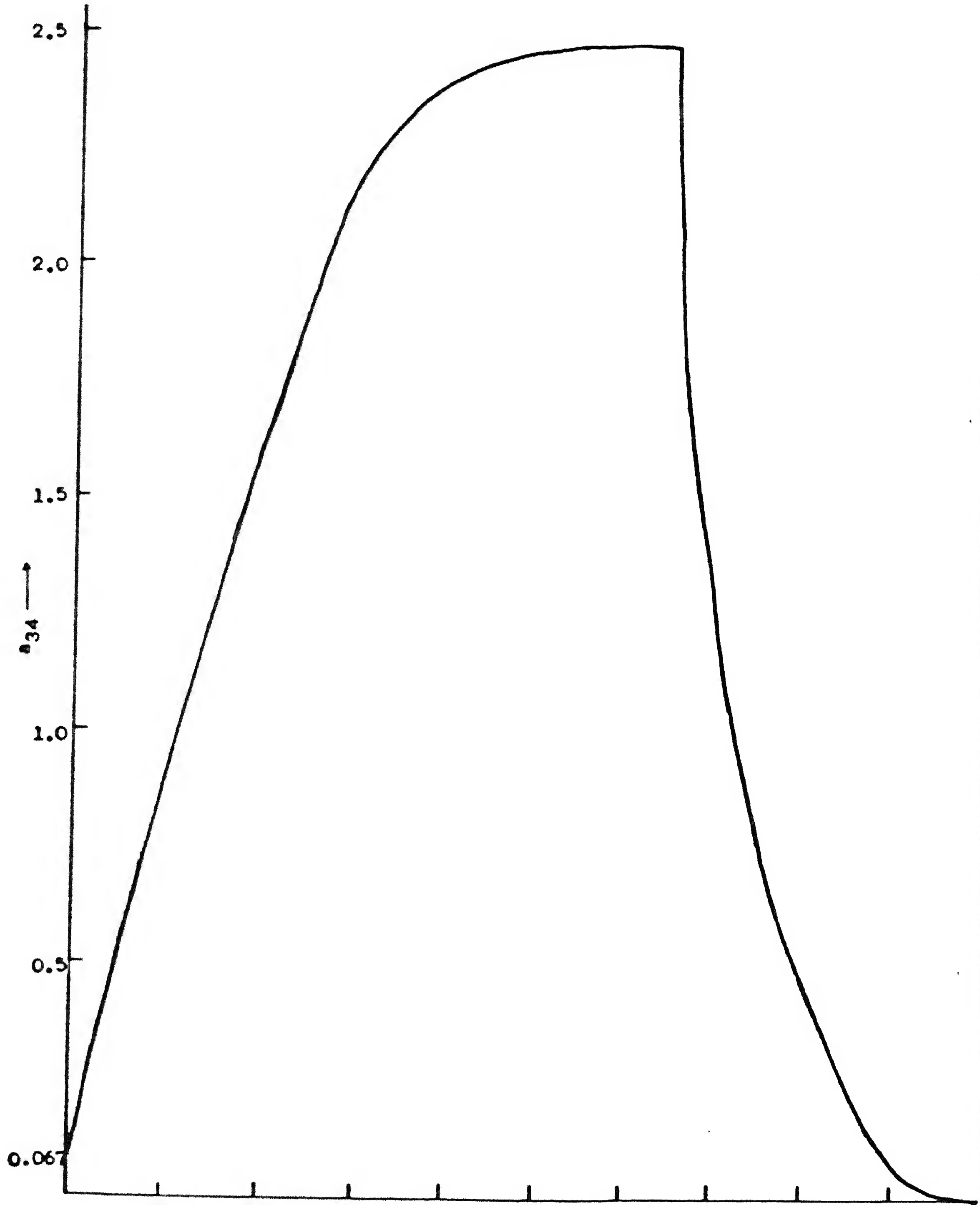


FIG. 3.3 PARAMETER a_{34} vs TIME PLOT

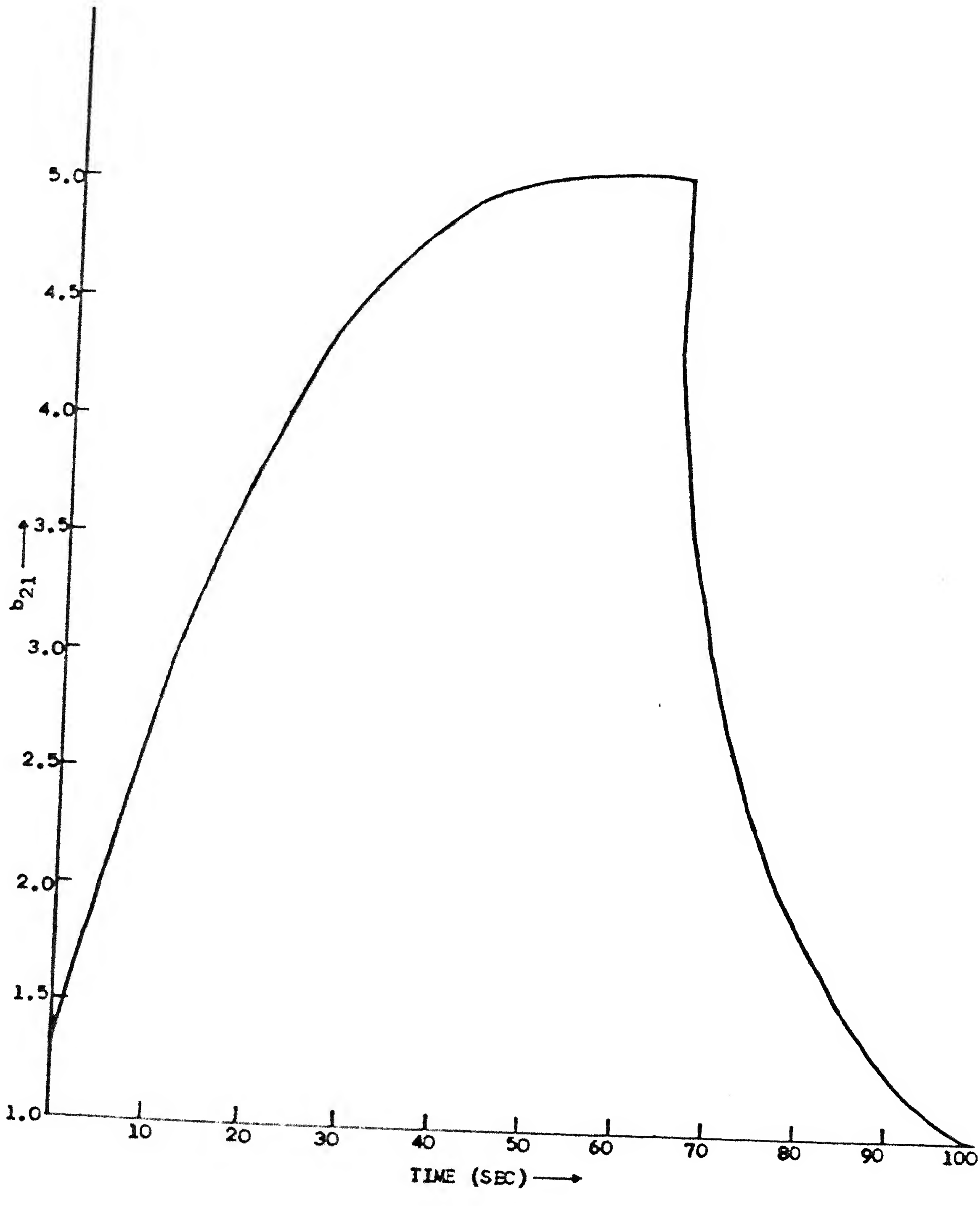
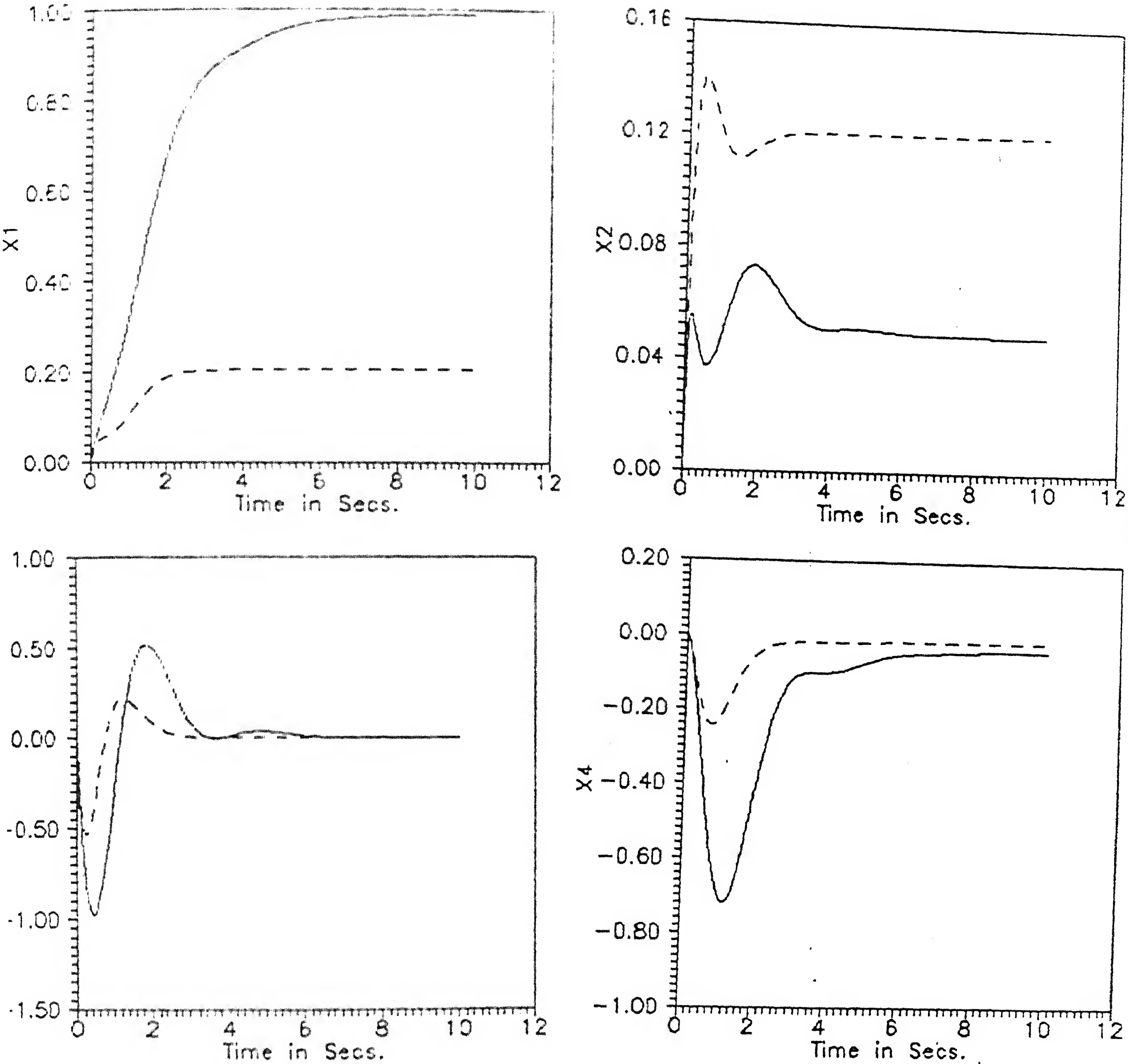


FIG. 3.4 PARAMETER b_{21} vs TIME PLOT

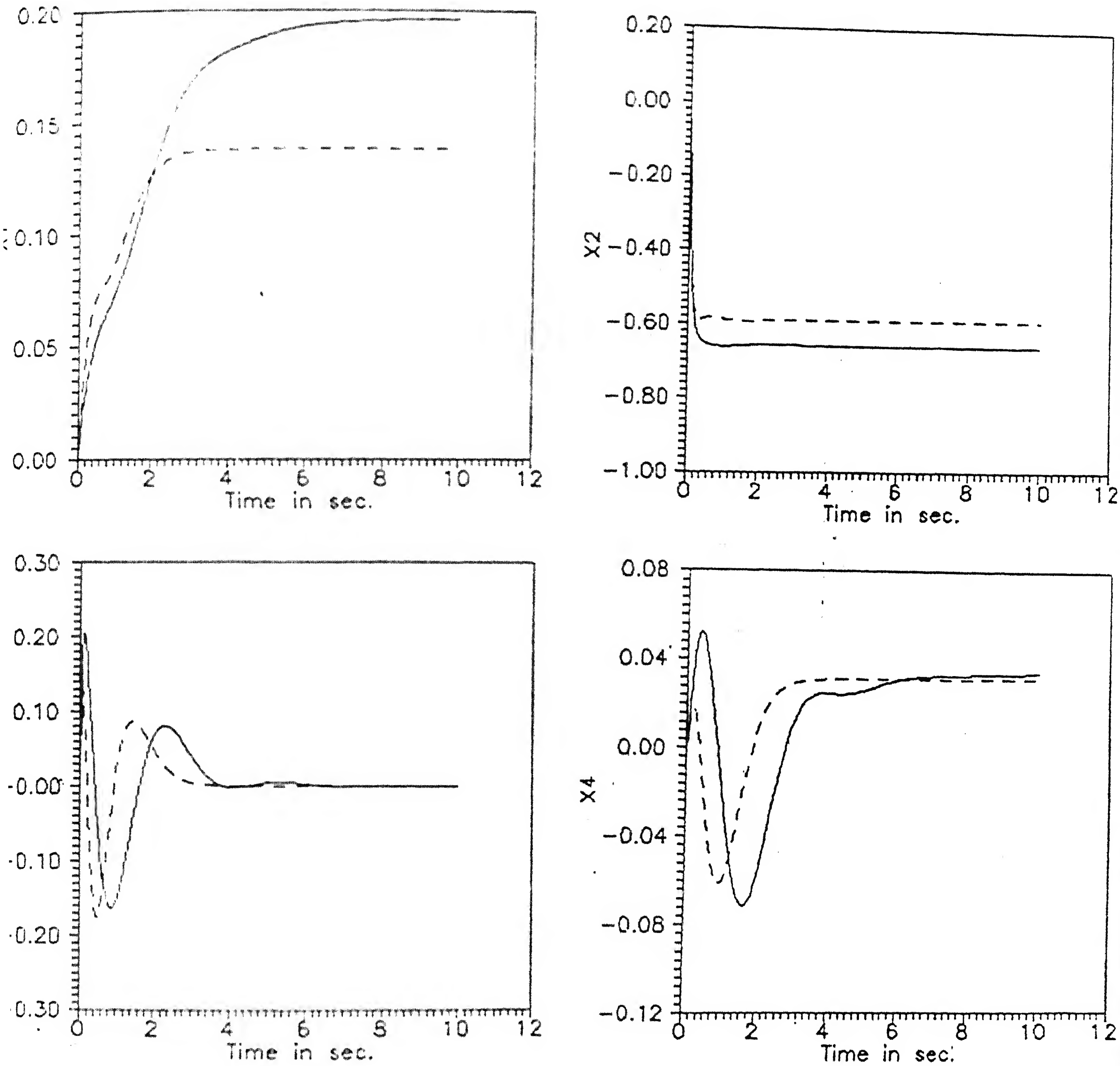


L^*_{60} —— Response of optimal controller

L_α ----- Response of designed controller

$\alpha = 0.8,\ u_1 = 1.0,\ u_2 = 0.0$

FIG. 3.5 STEP RESPONSE AT 60 KNOTS

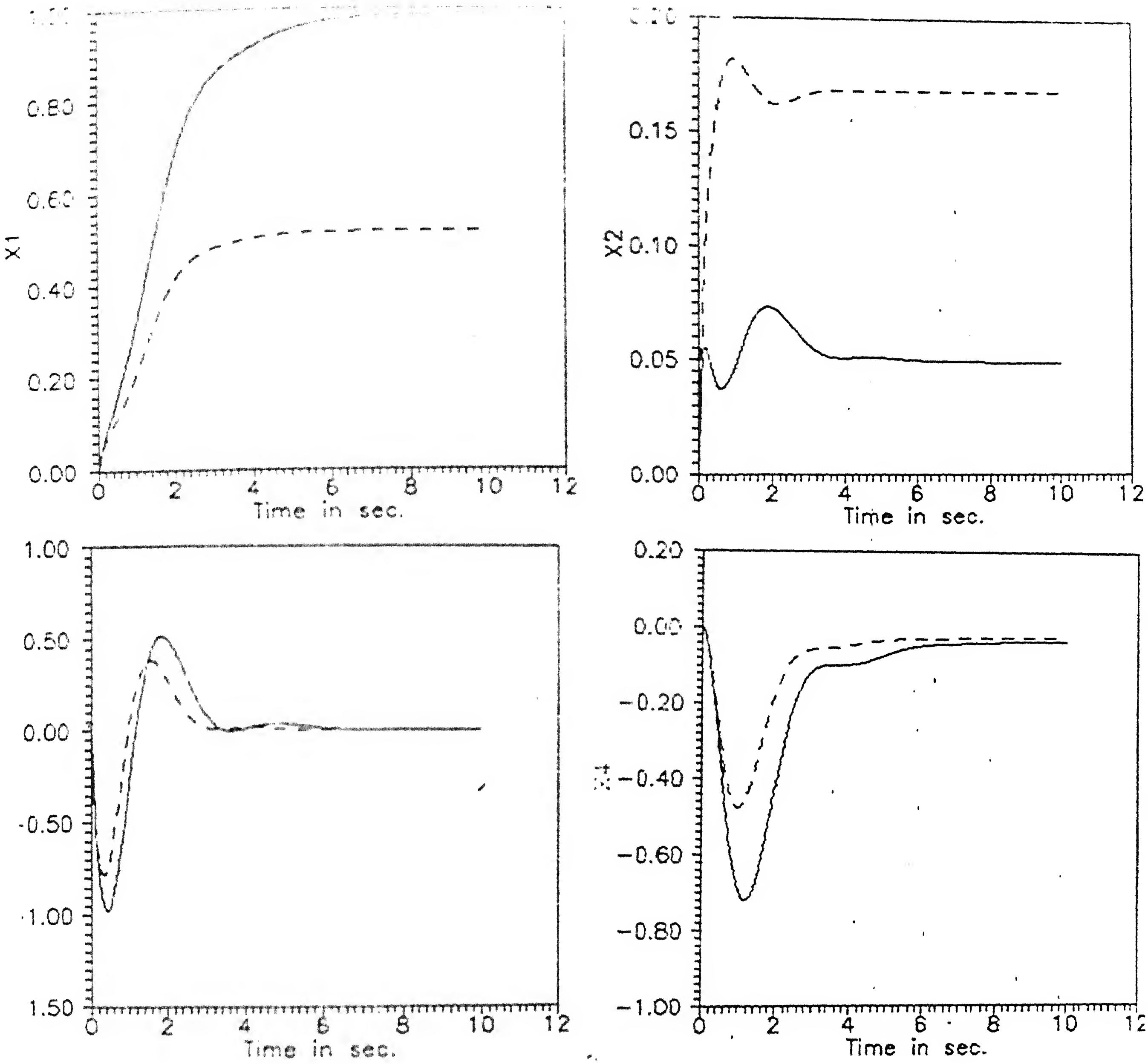


L^*_{60} ——— Response of optimal controller

L_α - - - - Response of designed controller

$= 0.8, \quad u_1 = 0.0, \quad u_2 = 1.0$

FIG. 3.6 STEP RESPONSE AT 60 KNOTS

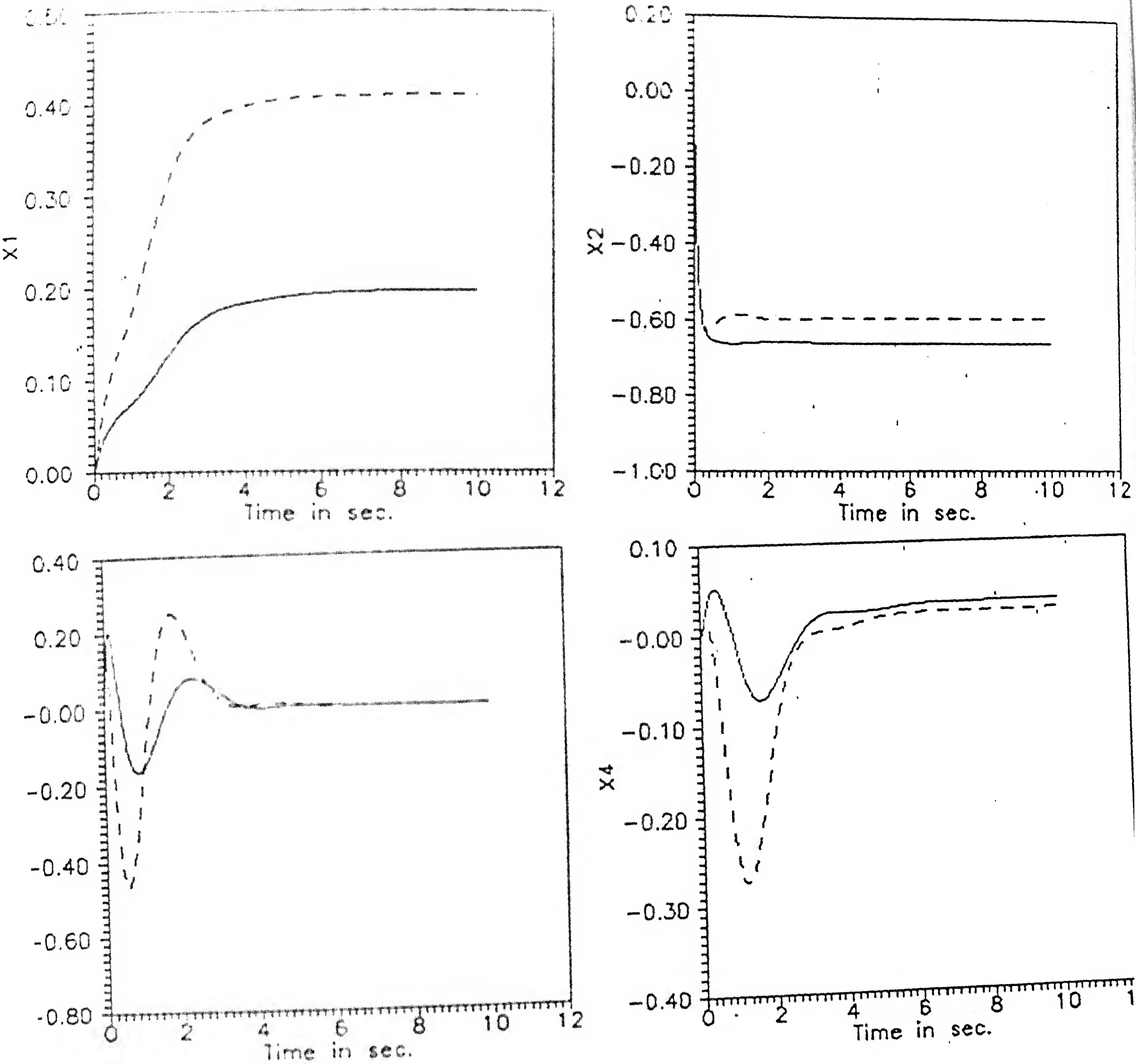


L^*_{60} —— Response of optimal controller

L_α ---- Response of designed controller

$\alpha = 0.3, \; u_1 = 1.0, \; u_2 = 0.0$

FIG. 3.7 STEP RESPONSE AT 60 KNOTS

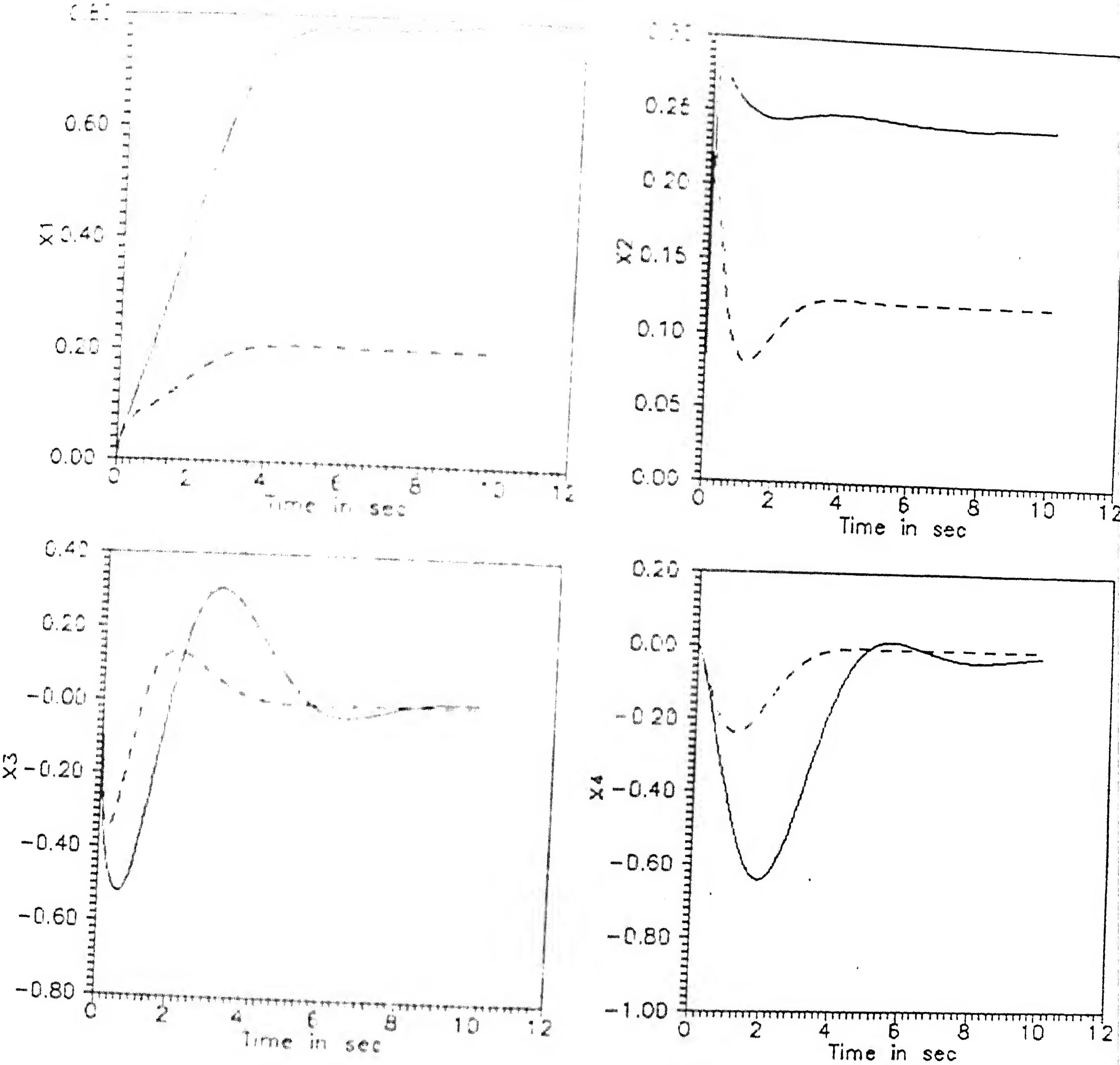


L^{*}_{60} —— Response of optimal controller

L_{α} ---- Response of designed controller

$\alpha = 0.3,\ u_1 = 0.0,\ u_2 = 1.0$

FIG. 3.8 STEP RESPONSE AT 60 KNOTS

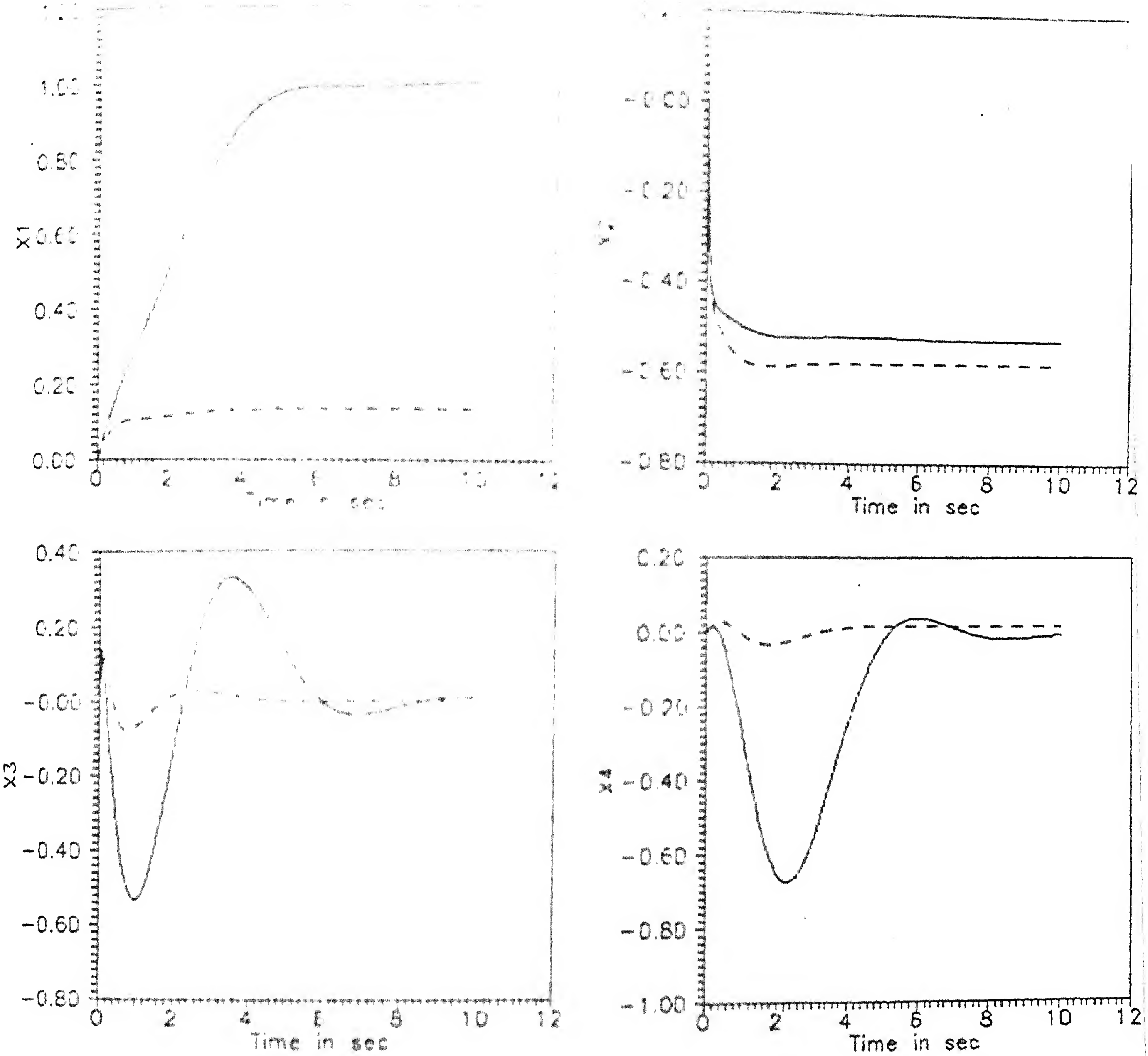


L^*_{170} ——— Response of optimal controller

L_α ---- Response of optimal controller

$\alpha = 0.8, \ u_1 = 1.0, \ u_2 = 0.0$

FIG. 3.9 STEP RESPONSE AT 170 KNOTS

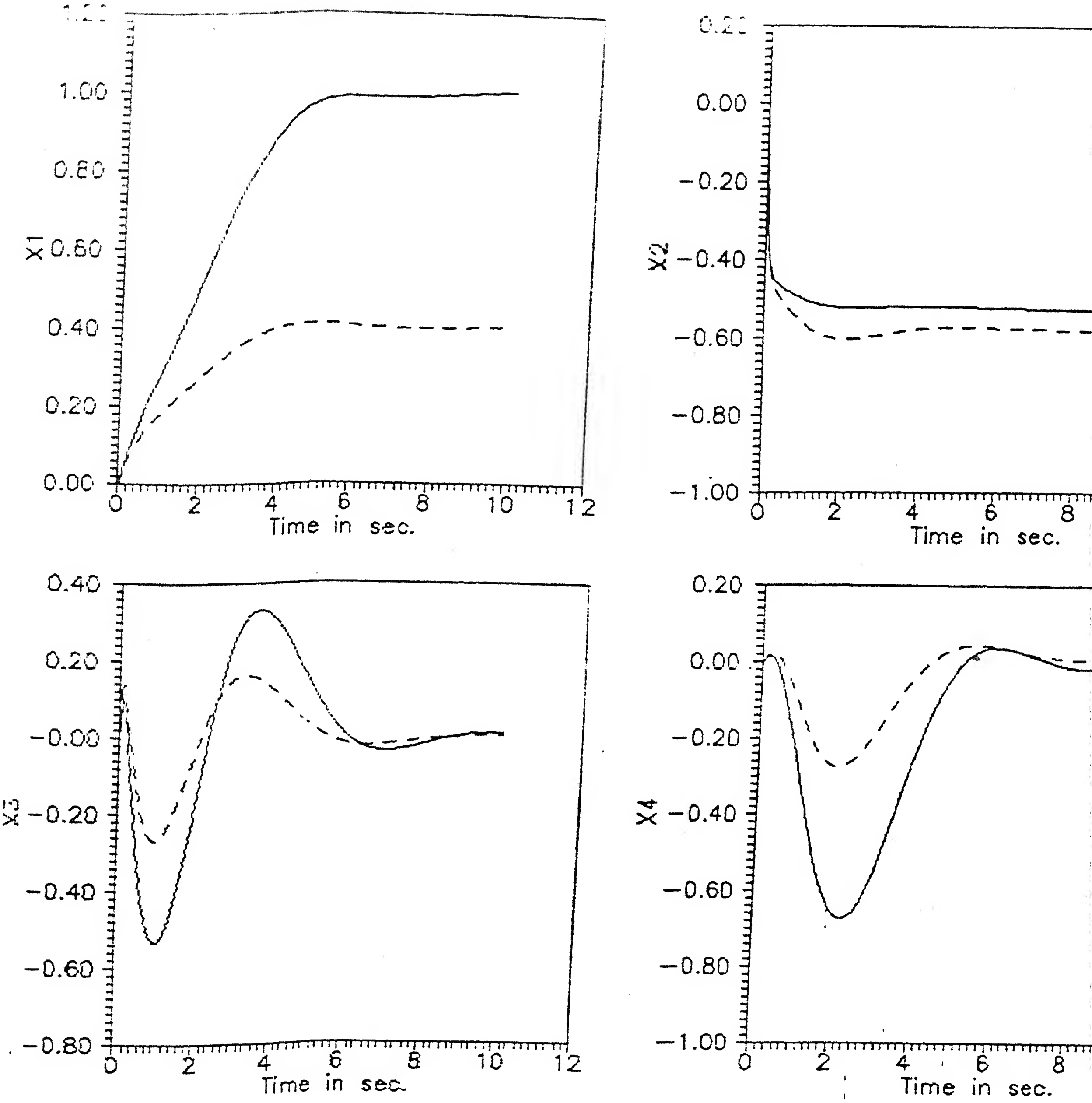


L^*_{170} ——— Response of optimal controller

$L_{\alpha 1}$ ---- Response of designed controller

$\alpha = 0.8, \ u_1 = 0.0, \ u_2 = 1.0$

FIG. 3.10 STEP RESPONSE AT 170 KNOTS

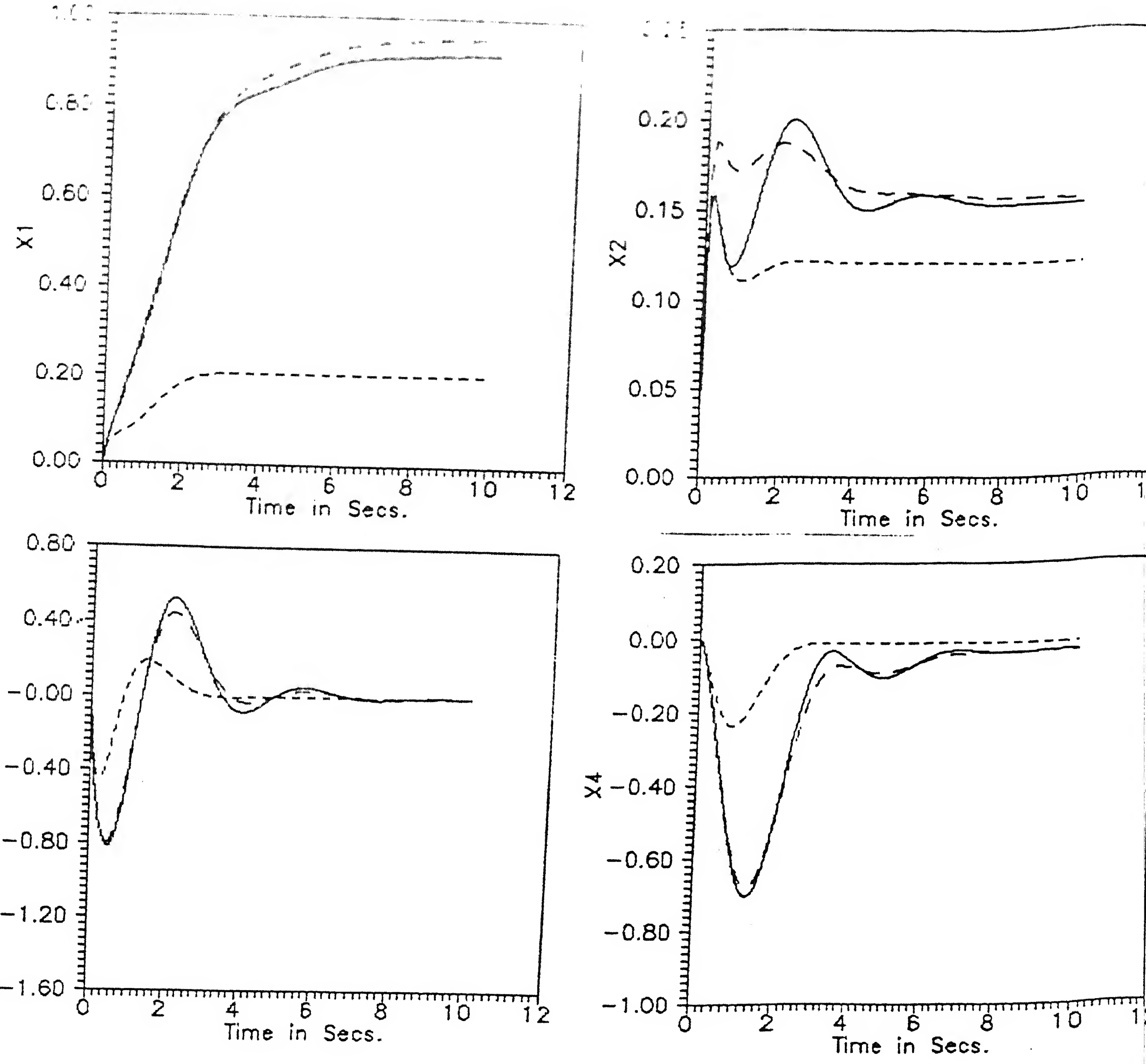


L^*_{170} —— Response of optimal controller

L_α ---- Response of designed controller

$\alpha = 0.3$, $u_1 = 0.0$, $u_2 = 1.0$

FIG. 3.12 STEP RESPONSE AT 170 KNOTS



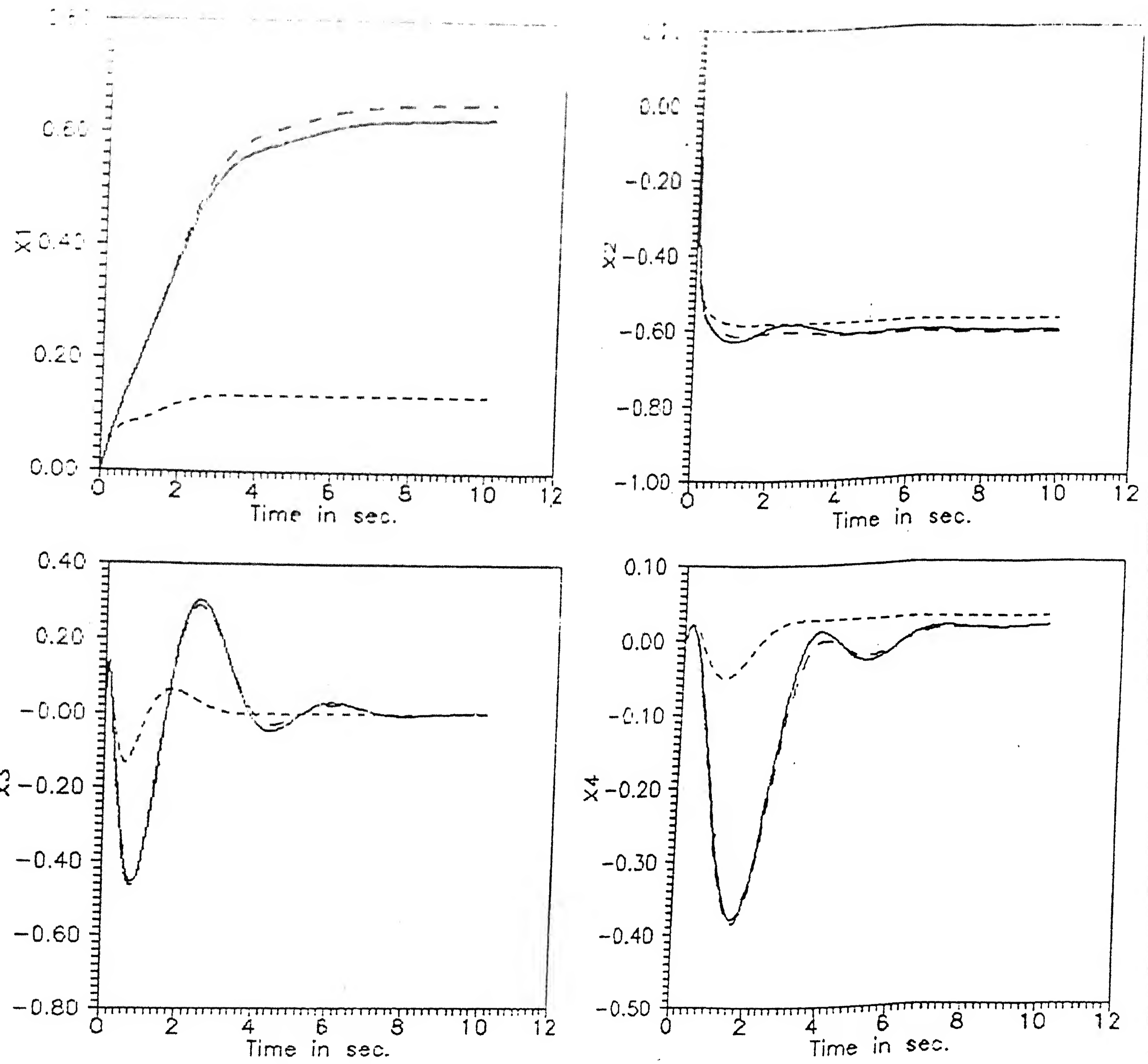
L^*_{115} ——— Response of optimal controller

L_α ---- Response of designed controller

L_{y115} − − − Response of approximate optimal controller

$\alpha = 0.8, \ u_1 = 1.0, \ u_2 = 0.0$

FIG. 3.13 STEP RESPONSE AT 115 KNOTS



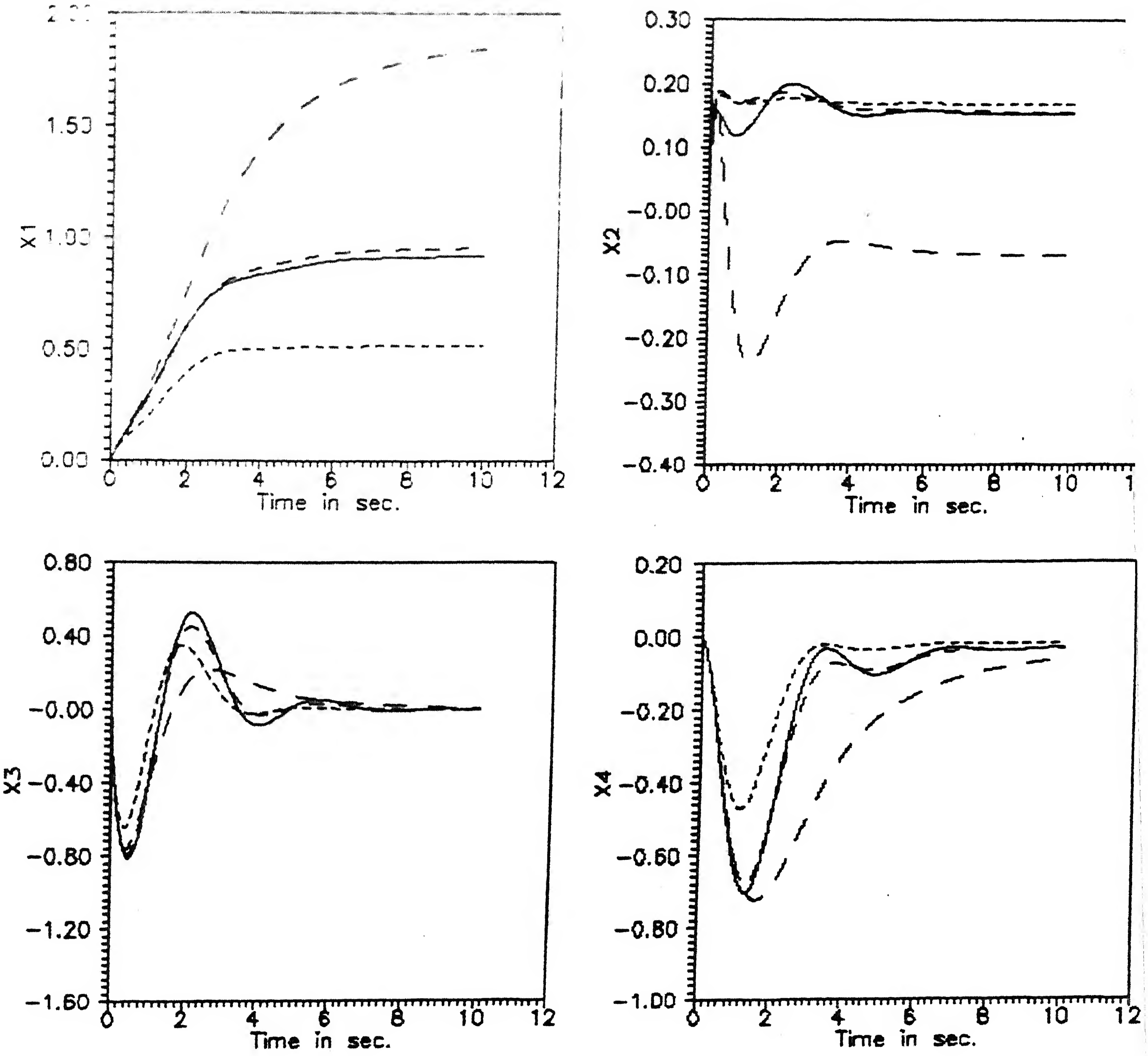
L^*_{115} ——— Response of optimal controller

L_α ---- Response of designed controller

L_{y115} — — — Response of approximate optimal controller

$\alpha = 0.8, \ u_1 = 0.0, \ u_2 = 1.0$

FIG. 3.14 STEP RESPONSE AT 115 KNOTS



L^*_{115} ——— Response of optimal controller

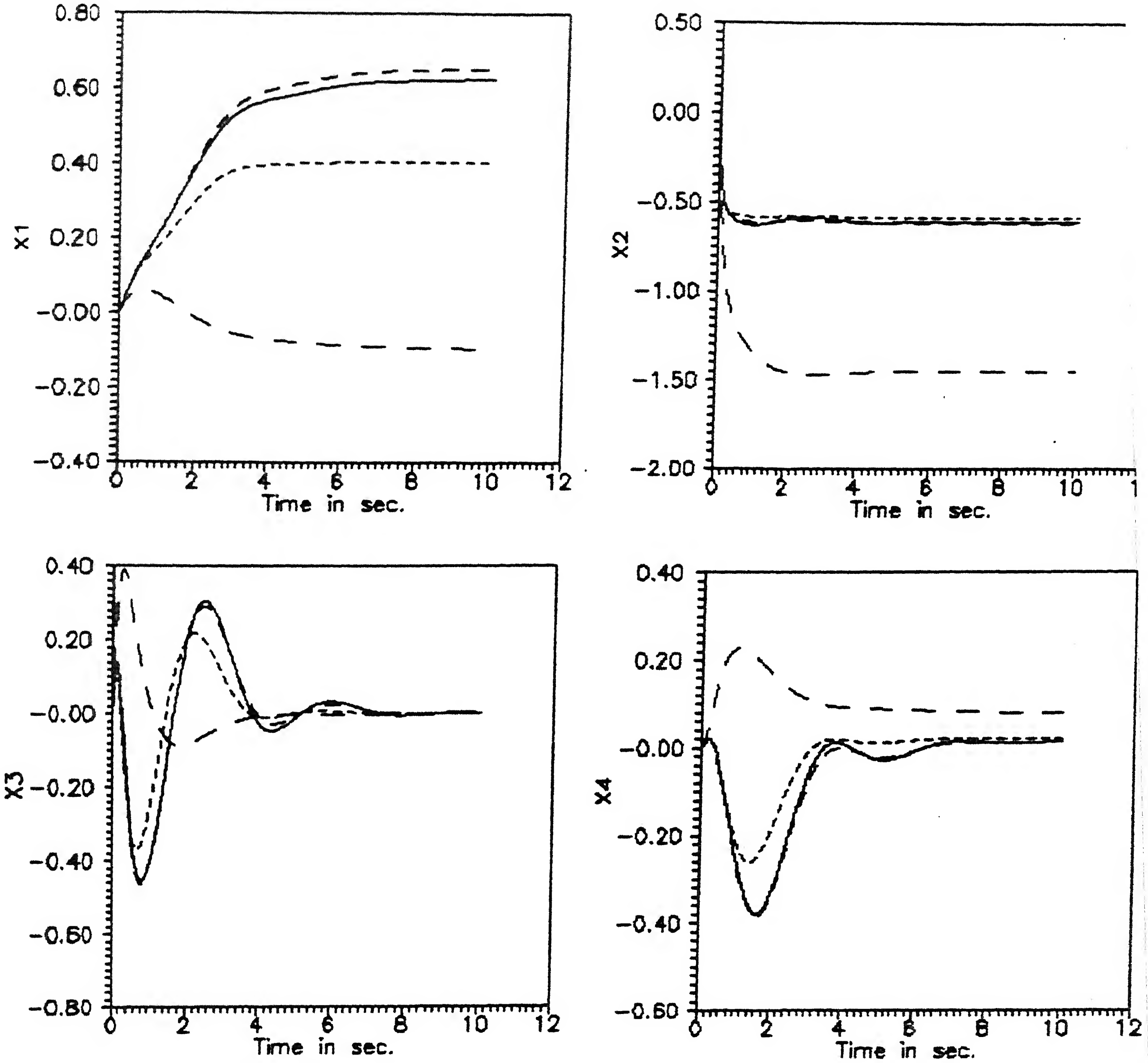
L_α ---- Response of designed controller

L_{y115} – – – Response of approximate optimal controller

L_{yd115} —— —— Response of Yedavalli and Liang's controller

$\alpha = 0.3, \ u_1 = 1.0, \ u_2 = 0.0$

FIG. 3.15 STEP RESPONSE AT 115 KNOTS



L^*_{115} ——— Response of optimal controller

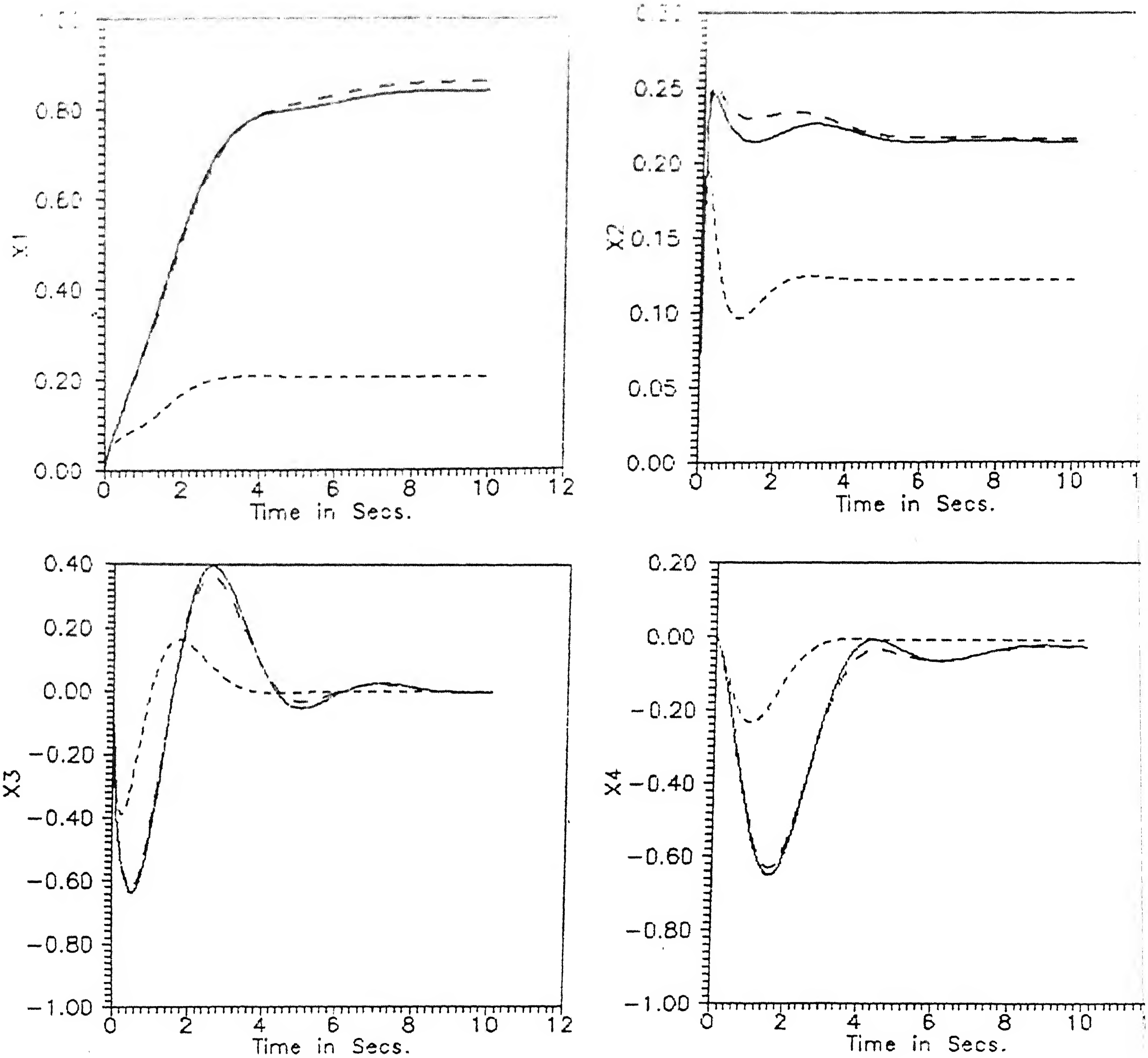
L_α ---- Response of designed controller

L_{y115} – – – Response of approximate optimal controller

L_{yd115} —— —— Response of Yedavalli and Liang's controller

$\alpha = 0.3$, $u_1 = 0.0$, $u_2 = 1.0$

FIG. 3.16 STEP RESPONSE AT 115 KNOTS



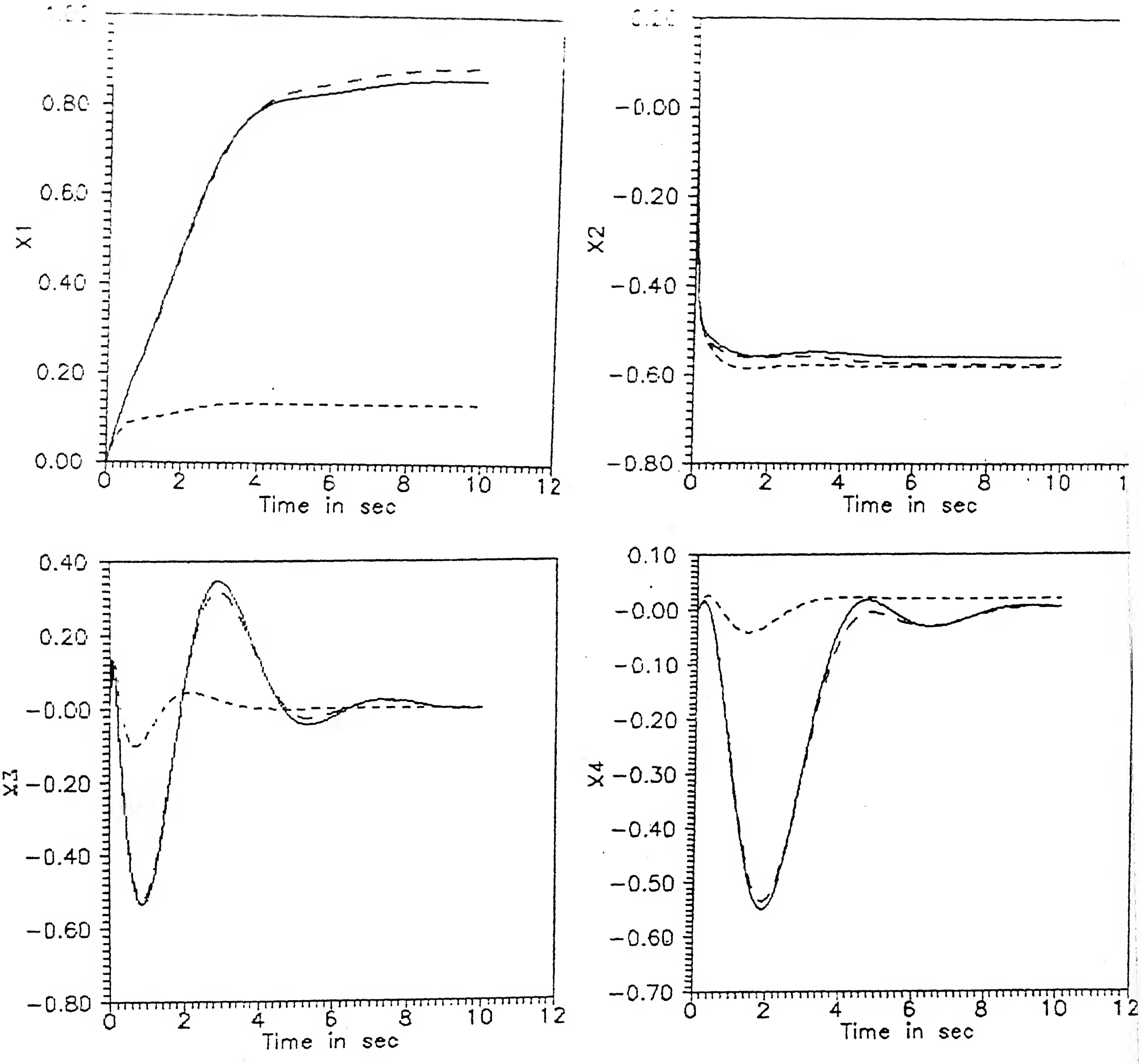
L^*_{150} ——— Response of optimal controller

L_α ---- Response of designed controller

$L_{y_{150}}$ _ _ Response of approximate optimal controller

$\alpha = 0.8, \ u_1 = 1.0, \ u_2 = 0.0$

FIG. 3.17 STEP RESPONSE AT 150 KNOTS



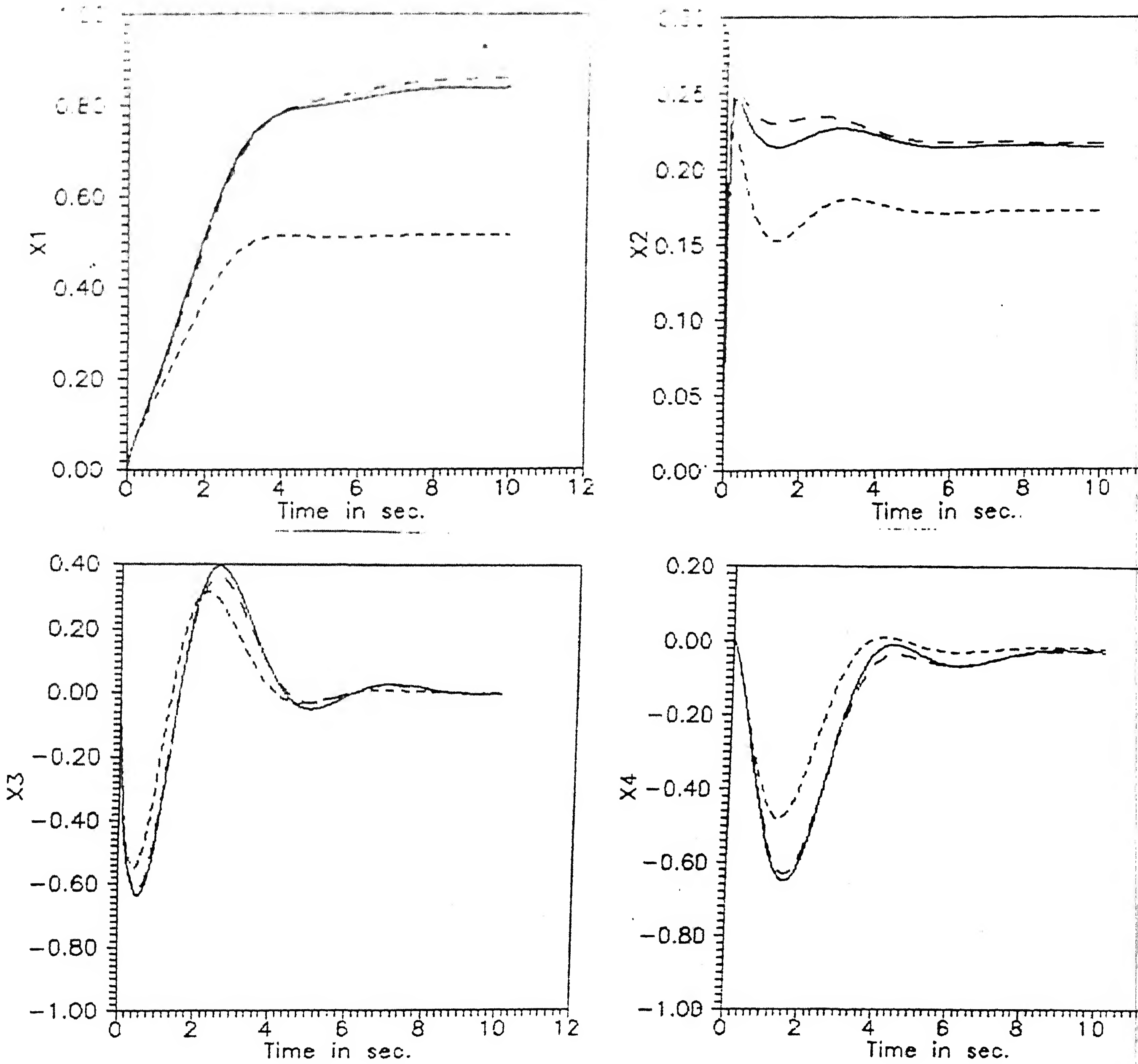
L^*_{150} ——— Response of optimal controller

L_α ---- Response of designed controller

$L_{y_{150}}$ _ _ Response of approximate optimal controller

$\alpha = 0.8,\ u_1 = 0.0\ u_2 = 1.0$

FIG. 3.18 STEP RESPONSE AT 150 KNOTS



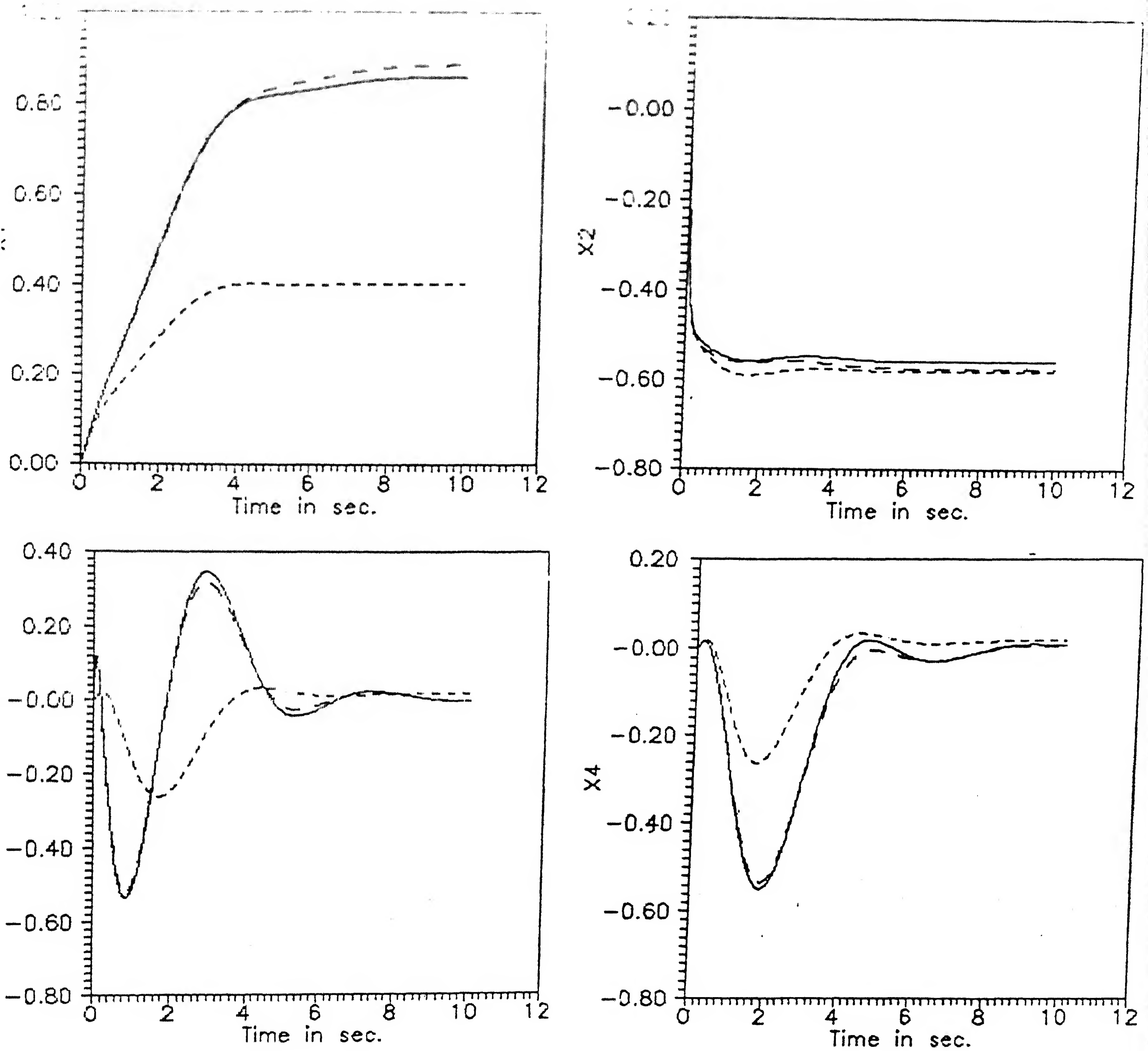
L^*_{150} —— Response of optimal controller

L_α ---- Response of designed controller

$L_{y_{150}}$ – – Response of approximate optimal controller

$\alpha = 0.3$, $u_1 = 1.0$, $u_2 = 0.0$

FIG. 3.19 STEP RESPONSE AT 150 KNOTS



L^*_{150} ——— Response of optimal controller

L_α ---- Response of designed controller

$L_{y_{150}}$ — — Response of approximate optimal controller

$\alpha = 0.3$, $u_1 = 0.0$, $u_2 = 1.0$

FIG. 3.20 STEP RESPONSE AT 150 KNOTS

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